Maple: An Introduction

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1 Introduction

This introductory article contains basic information for the new user of Maple who wants to spend only a couple of hours learning Maple. This document covers how to use Maple as a calculator, recovering from errors, Maples notation for arithmetic, mathematical constants and functions, how to get help, how to solve equations, do linear algebra, and to define and plot mathematical functions in two and three dimensions. For each section there are several examples. At the end there are some exercises which you can try to solve. Note, we have not attempted to show any of Maples programming facilities in this article.

2 Getting Started

You should be able to start Maple by typing `maple`. The Maple logo should appear almost immediately. After the Maple prompt `>` has appeared, Maple is ready to receive your input commands. You should see

```
    |\^/|      MAPLE V
._|\|   |/|_.  Copyright (c) 1981-1990 by the University of Waterloo.
 \  MAPLE  /   All rights reserved.  MAPLE is a registered trademark of
 <____ ____>   Waterloo Maple Software.
      |        Type ? for help.
>
```

Note, on workstations running X Windows, you can run a version of Maple which has a more sophisticated user interface called `xmaple`. Just type `xmaple` instead of `maple`. After a few seconds the Maple session window should appear which contains the Maple logo followed by the Maple prompt. You can now enter a command.

ATTENTION : every Maple command must end with a semicolon ;

This is a grammatical requirement of the Maple language. The semicolon tells Maple that this is the end of the input. Type now $\boxed{\texttt{4/6;}}$ followed by a return. Maple answers with

```
2/3
```

To refer to the latest result, one uses the double quote character ". Try it now

```
> ";
                        2/3
```

You can use the value of " in an expression as if it were a variable, for example

```
> 2*"+1;
                        7/3
```

In order to save results, one assigns them to a variable, e.g.

```
> R := ";
                     R := 7/3
```

If you want to interrupt a calculation push the interrupt key once. This is <Ctrl> C under Unix. When the prompt character appears, Maple is ready for further input. To exit Maple, you can type `quit`. Note, under `xmaple` you can click on the $\boxed{\text{Interrupt}}$ button to interrupt Maple and on the $\boxed{\text{Quit}}$ button to exit Maple.

3 Arithmetic and Maple Notation

The basic arithmetic operators and constants known to Maple are

Maple notation	Meaning	Mathematical notation
`x+y`	addition	$+$
`-x` and `x-y`	negation and subtraction	$-x$ and $x-y$
`x*y`	multiplication	xy
`x/y`	division	$\frac{x}{y}$
`x^y` or `x**y`	exponentiation	x^y
`k!`	factorial	$k!$
`I` or `sqrt(-1)`	complex unit	i or $\sqrt{-1}$
`Pi`		π
`infinity`		∞
`abs(x)`	absolute value	$\|x\|$
`sqrt(x)` or `x^(1/2)`	square root	$\sqrt{x}$
`exp(x)`	exponential function	e^x
`ln(x)` or `log(x)`	natural logarithm	$\ln x$
`sin(x)`	sine function	$\sin x$
`cos(x)`	cosine function	$\cos x$
`tan(x)`	tangent function	$\tan x$

Notice that Maple does arithmetic with integers exactly. I.e. exact arithmetic is used instead of decimal arithmetic. Use decimal numbers if you want decimal numbers. Examples

```
> 2*3+2/7;
                                    44/7

> 2.0*3.0+2.0/7;
                                 6.285714286
```

This principle works for formulae too. Use the evalf function if you want a decimal approximation.

```
>  sin(Pi/3);
                                        1/2
                                   1/2 3

>  evalf(");
                                 .8660254040
```

There is no limit on the length of integers in Maple. It is quite common to compute with integers several hundred digits long. It is also possible to do decimal arithmetic to more than the default 10 digits of precision. You can compute π to a 1000 digits if you want. This is done by assigning the the global variable Digits to the desired precision. Note, don't forget to reset Digits to 10 if you don't need more than 10 Digits! High precision operations take longer! Here are some examples

```
> 2^100;
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> Digits := 50:
> evalf( sin(Pi/3) );

             .86602540378443864676372317075293618347140262690520

> Digits := 10:
```

In the above examples we have used the colon to terminate a command. Use the colon : instead of a semicolon ; if you don't want to see the output.

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If you forget the semicolon don't panic! Maple will simply print another prompt indicating that it is still waiting for more input. Just type the semicolon and return and Maple will go ahead and compute the result. Note, this means that you can enter large expressions over one or more lines. Try to input a large expression like the following

```
> f := 4*x^4+3*x^3*y+2*x^2*y^2+
>      3*x*y^3+4*y^4;

                        4      3        2  2        3      4
              f := 4 x  + 3 x  y + 2 x  y  + 3 x y  + 4 y
```

If you incorrectly input an expression, Maple will respond with a *syntax error*. A common error is to forget the semicolon and then to try to correct the problem by retyping the input. For example, one user typed

```
> 2*x+1
> 2*x+1
```

and Maple replied

```
syntax error:
2*x+1
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```

The user forgot the semicolon after typing `2*x+1` and return. The user then typed in another `2*x+1`. Maple now says that there is an error and points to where it thinks the error is with the ^ character. What the user typed is equivalent to typing

```
> 2*x+1 2*x+1
```

Now you can see why Maple thinks the expression is wrong. To recover from a syntax error, just type a semicolon to clear the remaining input, and then reenter the command[1].

[1] In more sophisticated user interfaces, such as `xmaple`, you can edit your input

5 On-Line Help

If you need help for a Maple function, you can use the ? command. A ? by itself gives you general information. Try it now. E.g. ?min gives help on the min function.

```
FUNCTION: max - maximum of numbers
FUNCTION: min - minimum of numbers

CALLING SEQUENCE:
   max(x_1, x_2, ...)
   min( x_1, x_2, ...)

PARAMETERS:
   x_1, x_2, ... - any expressions

SYNOPSIS:
- The functions max and min return the maximum or minimum respectively of one
  or more arguments.

- Most often the arguments will be of type numeric, that is, integers, ration-
  als, or floats.  However, the functionality is more general, allowing any
  type of arguments for which an unevaluated function call may be returned.

EXAMPLES:
> max(3/2, 1.49);
                                    3/2
> min(3/2, 1.49);
                                    1.49
> max(3/5, evalf(ln(2)), 9/13);
                                .6931471806
> max(x+1, x+2, y);
                               max(x + 2, y)

SEE ALSO:  maximize, minimize
```

6 Simplification Commands

The output from Maple is not always in the simplest form. There are a number of commands such as `collect`, `combine`, `expand`, `factor` and `simplify` which can be used to simplify or rearrange the output into the desired form. For example, given the polynomial

```
> (x+y)*(x-y)-x^2;
                                                2
                             (x + y) (x - y) - x
```

if we multiply it out with the `expand` command the result is simpler

```
> expand(");
                                        2
                                     - y
```

Often factoring a polynomial yields a simpler result, e.g.

```
> x^4+x^2*y+2*x^2+2*x^3+2*x*y+2*x+y+1;

                  4    2         2      3
                 x  + x  y + 2 x  + 2 x  + 2 x y + 2 x + y + 1

> factor(");
                                    2   2
                             (x + 1)  (x  + y + 1)
```

The `simplify` command can simplify general expressions, e.g.

```
> simplify(exp(a+ln(b*exp(c))));

                                 b exp(a + c)
```

In this example the simplify command reduces a rational function

```
> (x^3-y^3)/(x^2+x*y+y^2);
                                    3    3
                                   x  - y
                                -------------
                                 2          2
                                x  + x y + y

> simplify(");
                                    x - y
```

7 Calculus Commands

Experiment with the integration, differentiation and summation commands. The Maple syntax for these functions and examples is as follows.

Maple notation	Meaning	Mathematical notation
`diff(f(x),x);`	Partial derivative	$\frac{d}{dx}f(x)$
`int(f(x),x);`	Indefinite integration	$\int f(x)dx$
`sum(f(x),x);`	Indefinite summation	$\sum_n f(n)$
`int(f(x),x=a..b);`	Definite integration	$\int_a^b f(x)dx$
`sum(f(k),k=a..b);`	Definite summation	$\sum_{k=a}^b f(k)$

```
> diff(ln(x),x);
                                     1/x

> diff(arcsin(a*x),x);
```

```
                              a
                       --------------
                             2  2 1/2
                       (1 - a  x )

> int(x*ln(x),x);
                        2               2
                   1/2 x  ln(x) - 1/4 x

> int(sqrt(1-x^2),x);
                              2 1/2
             1/2 x (1 - x )     + 1/2 arcsin(x)

> int(ln(x),x=1..2);
                       - 1 + 2 ln(2)

> sum(n^2,n);
                       3         2
                  1/3 n  - 1/2 n  + 1/6 n

> sum(a^k,k=1..n);
                       (n + 1)
                      a            a
                      -------- - -----
                        a - 1    a - 1
```

In the case of indefinite integration, you can test whether Maple's answer is right by differentiating the integral and subtracting in from the integrand. The difference should be zero! But Maple may not recognize that the difference is zero immediately. You may have to help Maple simplify the difference to 0. You can make use of `expand` and `simplify`. Example:

```
> f := (x^3+2*x^2-x)/(x^3-x^2+x-1);
                              3      2
                             x  + 2 x  - x
                       f := ---------------
                              3    2
                             x  - x  + x - 1

> int(f,x);
                                             2
                  x + ln(x - 1) + ln(x  + 1)

> diff(",x);
                            1           x
                       1 + ----- + 2 ------
                           x - 1      2
                                     x  + 1

> simplify("-f);
```

0

8 The Solve Commands

You can solve single equations and systems of linear and non-linear equations exactly with the `solve` command. For approximate solutions, use the `fsolve` command. For ordinary differential equations use the `dsolve` command. And for recurrence equations use the `rsolve` command. These commands have the following syntax.

`solve(e, x)`	solve the equation e for the unknown x
`solve({e1, ..., en}, {x1, ..., xm})`	solve the equations $e_1, \ldots, e_n$ for $x_1, \ldots, x_m$
`fsolve(e, x)`	solve the equation e numerically for x
`fsolve({e1, ..., en}, {x1, ..., xm})`	solve numerically $e_1, \ldots, e_n$ for $x_1, ..., x_m$
`dsolve(e, y(x))`	solve the ODE e for $y(x)$
`dsolve({e, i1, ..., in}, y(x))`	solve the ODE e and initial conditions $i_1, ..., i_n$ for $y(x)$
`rsolve(e, f(n))`	solve the recurrence equation e for $f(n)$
`rsolve({e, i1, ..., in}, f(n))`	solve the recurrence e given initial conditions $i_1, ..., i_n$ for $f(n)$

For example, to solve the equation $x^3 - 6x = 5$ for x

```
> solve( x^3-6*x=5, x );
                                       1/2              1/2
                      -1, 1/2 + 1/2 21    , 1/2 - 1/2 21
```

To solve the ODE $y(x) + y''(x) = e^x$ given initial conditions $y(0) = 1$ and $y'(0) = 0$

```
> dsolve({y(x)+diff(y(x),x$2)=exp(x), y(0)=1, D(y)(0)=0}, y(x));

                y(x) = 1/2 exp(x) - 1/2 sin(x) + 1/2 cos(x)
```

9 Other Commands and Functions

Maple knows about the elementary functions `ln, exp, sin, cos, tan,` etc. Some other special functions that you might need are

Maple notation	Function	Mathematical notation
`erf(x)`	Error function	$erf(x)$
`binomial(n,k)`	Binomial function	$\binom{n}{k}$
`GAMMA(x)`	Gamma function	$\Gamma(x)$
`Psi(x)`	Poly Gamma function	$\psi(x)$
`Zeta(x)`	Riemann Zeta function	$\zeta(x)$
`BesselJ(v,x)`	Bessel function of the first kind	$J_\nu(x)$

See also `?integer` for a list of functions for integers and `?polynom` for a list of commands for computing with polynomials. See `?mod` for a list of commands for computing over finite rings and fields.

10 Data Structures

Maple uses sequences, lists, sets, tables and arrays for representing more complicated data. See `?sequences`, `?list`, `?set`, `?table`, `?array` for more detailed help and examples. A sequence is a sequence of expressions separated by commas. The `seq` function is a very useful function for creating sequences. It's syntax is

```
seq( f(i), i=a..b )
```

Lists group sequences and are created using square brackets `[, ]`. Sets are like lists except duplicate entries are removed. Sets are created using squiggley brackets {, }. Examples

```
> S1 := 1,5,3;
                              S1 := 1, 5, 3

> max(S1);
                                    5

> S2 := seq(i^2, i=1..5);
                          S2 := 1, 4, 9, 16, 25

> S2[3];
                                    9

> S1 := {x,y,z,y};
                             S1 := {x, y, z}

> S1 union {w,x};
                               {x, y, z, w}

> L1 := [x,y,z,y];
                            L1 := [x, y, z, y]

> L2 := [L1[4],L1[2..3]];
                             L2 := [y, y, z]
```

11 The Linear Algebra Package

In Maple a vector is represented as a one-dimensional array, and a matrix is represented as a two-dimensional array. See `?vector` and `?matrix` for detailed help on vectors and matrices.

In Maple there are many packages for special applications. The `linalg` package contains many functions from linear algebra for computing with vectors and matrices. In order to use a package, you must load the package using the `with` command, e.g.

```
> with(linalg);
Warning: new definition for   norm
Warning: new definition for   trace
[BlockDiagonal, GramSchmidt, JordanBlock, add, addcol, addrow, adj, adjoint,
    angle, augment, backsub, band, basis, bezout, charmat, charpoly, col,
    coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod,
    curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals,
    eigenvects, equal, exponential, extend, ffgausselim, fibonacci, frobenius,
    gausselim, gaussjord, genmatrix, grad, hadamard, hermite, hessian,
    hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse,
    ismith, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve,
    matrix, minor, minpoly, mulcol, mulrow, multiply, norm, nullspace, orthog,
    permanent, pivot, potential, randmatrix, range, rank, row, rowdim,
    rowspace, rowspan, rref, scalarmul, singularvals, smith, stack, submatrix,
    subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace,
    transpose, vandermonde, vecpotent, vectdim, vector]
```

You can now use any of the functions listed. The `matrix` command can be used to input a matrix. In the next example the inverse and determinant of a 3 by 3 matrix is computed.

```
> A := matrix([[x-1,2,3],[0,x-2,2],[2,1,x-3]]);
```

$$A := \begin{pmatrix} x-1 & 2 & 3 \\ 0 & x-2 & 2 \\ 2 & 1 & x-3 \end{pmatrix}$$

```
> det(A);
```

$$x^3 - 6x^2 + 3x + 16$$

```
> inverse(A);
```

$$\begin{pmatrix} \frac{x^2-5x+4}{x^3-6x^2+3x+16} & -\frac{2x-9}{x^3-6x^2+3x+16} & -\frac{3x-10}{x^3-6x^2+3x+16} \\ \frac{4}{x^3-6x^2+3x+16} & \frac{x^2-4x-3}{x^3-6x^2+3x+16} & -\frac{2x-2}{x^3-6x^2+3x+16} \\ -\frac{2x-4}{x^3-6x^2+3x+16} & -\frac{x-5}{x^3-6x^2+3x+16} & \frac{(x-1)(x-2)}{x^3-6x^2+3x+16} \end{pmatrix}$$

If you type `?packages` you will get a list of all the known packages to Maple and what they are. In particular, this includes

Package	Explanation
`numtheory`	number theoretic functions
`combinat`	combinatorial functions
`stats`	statistical functions
`geometry`	functions for dealing with two dimensional euclidean geometry
`orthopoly`	definition of orthogonal polynomials
`simplex`	functions for linear optimization problems

12 Functions and Graphics

Mathematical functions of one or more variables can be defined in Maple. The function can be evaluated at either numerical or symbolic expressions. For example

```
> f:=x->sin(x)/x;
                              sin(x)
                    f := x -> ------
                                x

> f(2.0);
                         .4546487134

> f(t);
                           sin(t)
                           ------
                             t
```

Functions can be graphed with the `plot` command which has the following syntax.

```
plot( f, a..b )
```

That means, that the function f will be drawn on the interval a to b. Try this now

```
> plot(f,-12..12);
```

An example of a function in two variables

```
> g:=(x,y)->(x^2-y^2)/(x^2+y^2);
                               2    2
                              x  - y
                 g := (x,y) -> -------
                               2    2
                              x  + y

> g(1,2);
                            -3/5

> g(1,x);
```

```
       2
1 - x
------
       2
1 + x
```

Functions of two variables can be graphed with the `plot3d` command

```
plot3d( f, a..b, c..d )
```

```
> plot3d(g, -1..1, -1..1);
```

The examples above are plots of functions. The other possibility is to graph an *expression*. The syntax and examples for plotting expressions is

```
plot( f(x), x=a..b )
plot3d( f(x,y), x=a..b, y=c..d )
```

```
> plot( sin(x)/x, x=-12..12 );
> plot3d( (x^2-y^2)/(x^2+y^2), x=-1..1, y=-1..1 );
```

13 Exercises

1. Calculate the first and second derivative of $\sin(x)\cos(x)$ wrt x.

2. Given the polynomial $y(x) = x^3 - 4x^2 + 4x - 1$ find the roots and any local minima and maxima of $y(x)$. Check your solutions with a plot of the polynomial.

3. Given $f = x^2 - 4$ calculate the integral of f and $\frac{1}{f}$ with respect to x. Check that Maples answers are correct by differentiating the results.

4. Compute the following integrals

$$\int_0^\infty e^{-t}\mathrm{d}t \qquad \text{and} \qquad \int_0^\infty e^{-t^2}\mathrm{d}t$$

5. Compute the following sums

$$\sum_{k=1}^{1000} k \qquad \text{and} \qquad \sum_{k=1}^{\infty} 1/k^2$$

6. Compute a formula for the sum of the first n integers and also the sum of the squares of the first n integers. I.e. compute and simplify the sums

$$\sum_{k=1}^{n} k \qquad \text{and} \qquad \sum_{k=1}^{n} k^2$$

7. Given the function $h(x) = 1 - x + sin(x)$, define the function in Maple and compute the value of the function h at $x = -2.0$ and then graph the function in the region $[-5,5]$.

8. Input the following Matrix A into Maple

$$A = \begin{pmatrix} a & 0 & 5 \\ 1 & 1 & 1 \\ -a & 0 & 0 \end{pmatrix}$$

   (a) Compute the characteristic polynomial of A. Hint: use the `charmat` and `det` commands in the linear algebra package `linalg`.

   (b) Compute the eigenvalues of A. Hint: use the `solve` or `factor` commands to find the roots of the characteristic polynomial.

9. Use the `solve` command to solve the linear system

$$\begin{vmatrix} 4x - 5y = 11 \\ 2x + y = 9 \end{vmatrix}$$

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`x^y` or `x**y`	exponentiation	x^y
`k!`	factorial	$k!$
`I` or `sqrt(-1)`	complex unit	i or $\sqrt{-1}$
`Pi`		π
`infinity`		∞
`abs(x)`	absolute value	$\|x\|$
`sqrt(x)` or `x^(1/2)`	square root	$\sqrt{x}$
`exp(x)`	exponential function	e^x
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> 2*3+2/7;
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This principle works for formulae too. Use the `evalf` function if you want a decimal approximation.

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                                          1/2
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```
> f := 4*x^4+3*x^3*y+2*x^2*y^2+
>      3*x*y^3+4*y^4;

                       4      3        2  2        3      4
             f := 4 x  + 3 x  y + 2 x  y  + 3 x y  + 4 y
```

If you incorrectly input an expression, Maple will respond with a *syntax error*. A common error is to forget the semicolon and then to try to correct the problem by retyping the input. For example, one user typed

```
> 2*x+1
> 2*x+1
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and Maple replied

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```

Now you can see why Maple thinks the expression is wrong. To recover from a syntax error, just type a semicolon to clear the remaining input, and then reenter the command[1].

[1] In more sophisticated user interfaces, such as `xmaple`, you can edit your input

5 On-Line Help

If you need help for a Maple function, you can use the `?` command. A `?` by itself gives you general information. Try it now. E.g. `?min` gives help on the `min` function.

```
FUNCTION: max - maximum of numbers
FUNCTION: min - minimum of numbers

CALLING SEQUENCE:
   max(x_1, x_2, ...)
   min( x_1, x_2, ...)

PARAMETERS:
   x_1, x_2, ... - any expressions

SYNOPSIS:
- The functions max and min return the maximum or minimum respectively of one
  or more arguments.

- Most often the arguments will be of type numeric, that is, integers, ration-
  als, or floats.  However, the functionality is more general, allowing any
  type of arguments for which an unevaluated function call may be returned.

EXAMPLES:
> max(3/2, 1.49);
                                    3/2
> min(3/2, 1.49);
                                    1.49
> max(3/5, evalf(ln(2)), 9/13);
                                .6931471806
> max(x+1, x+2, y);
                               max(x + 2, y)

SEE ALSO:  maximize, minimize
```

6 Simplification Commands

The output from Maple is not always in the simplest form. There are a number of commands such as `collect`, `combine`, `expand`, `factor` and `simplify` which can be used to simplify or rearrange the output into the desired form. For example, given the polynomial

```
> (x+y)*(x-y)-x^2;
                                               2
                              (x + y) (x - y) - x
```

if we multiply it out with the `expand` command the result is simpler

```
> expand(");
                                            2
                                         - y
```

Often factoring a polynomial yields a simpler result, e.g.

```
> x^4+x^2*y+2*x^2+2*x^3+2*x*y+2*x+y+1;

                   4    2         2      3
                  x  + x  y + 2 x  + 2 x  + 2 x y + 2 x + y + 1

> factor(");
                                       2   2
                               (x + 1)  (x  + y + 1)
```

The `simplify` command can simplify general expressions, e.g.

```
> simplify(exp(a+ln(b*exp(c))));

                                  b exp(a + c)
```

In this example the simplify command reduces a rational function

```
> (x^3-y^3)/(x^2+x*y+y^2);
                                      3    3
                                     x  - y
                                  -------------
                                   2          2
                                  x  + x y + y

> simplify(");
                                      x - y
```

7 Calculus Commands

Experiment with the integration, differentiation and summation commands. The Maple syntax for these functions and examples is as follows.

Maple notation	Meaning	Mathematical notation
`diff(f(x),x);`	Partial derivative	$\frac{d}{dx}f(x)$
`int(f(x),x);`	Indefinite integration	$\int f(x)dx$
`sum(f(x),x);`	Indefinite summation	$\sum_n f(n)$
`int(f(x),x=a..b);`	Definite integration	$\int_a^b f(x)dx$
`sum(f(k),k=a..b);`	Definite summation	$\sum_{k=a}^b f(k)$

```
> diff(ln(x),x);
                                       1/x

> diff(arcsin(a*x),x);
```

```
                       a
                 --------------
                       2  2 1/2
                 (1 - a  x )
```

```
> int(x*ln(x),x);
                        2               2
                 1/2 x  ln(x) - 1/4 x

> int(sqrt(1-x^2),x);
                           2 1/2
              1/2 x (1 - x )    + 1/2 arcsin(x)

> int(ln(x),x=1..2);
                       - 1 + 2 ln(2)

> sum(n^2,n);
                       3         2
                 1/3 n  - 1/2 n  + 1/6 n

> sum(a^k,k=1..n);
                      (n + 1)
                     a            a
                     -------- - -----
                      a - 1     a - 1
```

In the case of indefinite integration, you can test whether Maple's answer is right by differentiating the integral and subtracting in from the integrand. The difference should be zero! But Maple may not recognize that the difference is zero immediately. You may have to help Maple simplify the difference to 0. You can make use of `expand` and `simplify`. Example:

```
> f := (x^3+2*x^2-x)/(x^3-x^2+x-1);
                           3      2
                          x  + 2 x  - x
                    f := ---------------
                          3    2
                         x  - x  + x - 1

> int(f,x);
                                            2
                  x + ln(x - 1) + ln(x  + 1)

> diff(",x);
                          1          x
                   1 + ----- + 2 ------
                       x - 1      2
                                 x  + 1

> simplify("-f);
```

0

8 The Solve Commands

You can solve single equations and systems of linear and non-linear equations exactly with the `solve` command. For approximate solutions, use the `fsolve` command. For ordinary differential equations use the `dsolve` command. And for recurrence equations use the `rsolve` command. These commands have the following syntax.

`solve(e, x)`	solve the equation e for the unknown x
`solve({e1, ..., en}, {x1, ..., xm})`	solve the equations $e_1, \ldots, e_n$ for $x_1, \ldots, x_m$
`fsolve(e, x)`	solve the equation e numerically for x
`fsolve({e1, ..., en}, {x1, ..., xm})`	solve numerically $e_1, \ldots, e_n$ for $x_1, ..., x_m$
`dsolve(e, y(x))`	solve the ODE e for $y(x)$
`dsolve({e, i1, ..., in}, y(x))`	solve the ODE e and initial conditions $i_1, ..., i_n$ for $y(x)$
`rsolve(e, f(n))`	solve the recurrence equation e for $f(n)$
`rsolve({e, i1, ..., in}, f(n))`	solve the recurrence e given initial conditions $i_1, ..., i_n$ for $f(n)$

For example, to solve the equation $x^3 - 6x = 5$ for x

```
> solve( x^3-6*x=5, x );
                                        1/2              1/2
                         -1, 1/2 + 1/2 21    , 1/2 - 1/2 21
```

To solve the ODE $y(x) + y''(x) = e^x$ given initial conditions $y(0) = 1$ and $y'(0) = 0$

```
> dsolve({y(x)+diff(y(x),x$2)=exp(x), y(0)=1, D(y)(0)=0}, y(x));

                y(x) = 1/2 exp(x) - 1/2 sin(x) + 1/2 cos(x)
```

9 Other Commands and Functions

Maple knows about the elementary functions `ln, exp, sin, cos, tan,` etc. Some other special functions that you might need are

Maple notation	Function	Mathematical notation
`erf(x)`	Error function	$erf(x)$
`binomial(n,k)`	Binomial function	$\binom{n}{k}$
`GAMMA(x)`	Gamma function	$\Gamma(x)$
`Psi(x)`	Poly Gamma function	$\psi(x)$
`Zeta(x)`	Riemann Zeta function	$\zeta(x)$
`BesselJ(v,x)`	Bessel function of the first kind	$J_\nu(x)$

See also `?integer` for a list of functions for integers and `?polynom` for a list of commands for computing with polynomials. See `?mod` for a list of commands for computing over finite rings and fields.

10 Data Structures

Maple uses sequences, lists, sets, tables and arrays for representing more complicated data. See `?sequences`, `?list`, `?set`, `?table`, `?array` for more detailed help and examples. A sequence is a sequence of expressions separated by commas. The `seq` function is a very useful function for creating sequences. It's syntax is

```
seq( f(i), i=a..b )
```

Lists group sequences and are created using square brackets `[, ]`. Sets are like lists except duplicate entries are removed. Sets are created using squiggley brackets {, }. Examples

```
> S1 := 1,5,3;
                              S1 := 1, 5, 3

> max(S1);
                                    5

> S2 := seq(i^2, i=1..5);
                          S2 := 1, 4, 9, 16, 25

> S2[3];
                                    9

> S1 := {x,y,z,y};
                             S1 := {x, y, z}

> S1 union {w,x};
                               {x, y, z, w}

> L1 := [x,y,z,y];
                            L1 := [x, y, z, y]

> L2 := [L1[4],L1[2..3]];
                             L2 := [y, y, z]
```

11 The Linear Algebra Package

In Maple a vector is represented as a one-dimensional array, and a matrix is represented as a two-dimensional array. See `?vector` and `?matrix` for detailed help on vectors and matrices.

In Maple there are many packages for special applications. The `linalg` package contains many functions from linear algebra for computing with vectors and matrices. In order to use a package, you must load the package using the `with` command, e.g.

```
> with(linalg);
Warning: new definition for   norm
Warning: new definition for   trace
[BlockDiagonal, GramSchmidt, JordanBlock, add, addcol, addrow, adj, adjoint,
    angle, augment, backsub, band, basis, bezout, charmat, charpoly, col,
    coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod,
    curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals,
    eigenvects, equal, exponential, extend, ffgausselim, fibonacci, frobenius,
    gausselim, gaussjord, genmatrix, grad, hadamard, hermite, hessian,
    hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse,
    ismith, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve,
    matrix, minor, minpoly, mulcol, mulrow, multiply, norm, nullspace, orthog,
    permanent, pivot, potential, randmatrix, range, rank, row, rowdim,
    rowspace, rowspan, rref, scalarmul, singularvals, smith, stack, submatrix,
    subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace,
    transpose, vandermonde, vecpotent, vectdim, vector]
```

You can now use any of the functions listed. The `matrix` command can be used to input a matrix. In the next example the inverse and determinant of a 3 by 3 matrix is computed.

```
> A := matrix([[x-1,2,3],[0,x-2,2],[2,1,x-3]]);
```

$$A := \begin{pmatrix} x-1 & 2 & 3 \\ 0 & x-2 & 2 \\ 2 & 1 & x-3 \end{pmatrix}$$

```
> det(A);
```

$$x^3 - 6x^2 + 3x + 16$$

```
> inverse(A);
```

$$\begin{pmatrix} \frac{x^2-5x+4}{x^3-6x^2+3x+16} & -\frac{2x-9}{x^3-6x^2+3x+16} & -\frac{3x-10}{x^3-6x^2+3x+16} \\ \frac{4}{x^3-6x^2+3x+16} & \frac{x^2-4x-3}{x^3-6x^2+3x+16} & -\frac{2x-2}{x^3-6x^2+3x+16} \\ -\frac{2x-4}{x^3-6x^2+3x+16} & -\frac{x-5}{x^3-6x^2+3x+16} & \frac{(x-1)(x-2)}{x^3-6x^2+3x+16} \end{pmatrix}$$

If you type `?packages` you will get a list of all the known packages to Maple and what they are. In particular, this includes

Package	Explanation
`numtheory`	number theoretic functions
`combinat`	combinatorial functions
`stats`	statistical functions
`geometry`	functions for dealing with two dimensional euclidean geometry
`orthopoly`	definition of orthogonal polynomials
`simplex`	functions for linear optimization problems

12 Functions and Graphics

Mathematical functions of one or more variables can be defined in Maple. The function can be evaluated at either numerical or symbolic expressions. For example

```
> f:=x->sin(x)/x;
                                 sin(x)
                       f := x -> ------
                                   x

> f(2.0);
                          .4546487134

> f(t);
                             sin(t)
                             ------
                               t
```

Functions can be graphed with the `plot` command which has the following syntax.

$$\boxed{\texttt{plot(f, a..b)}}$$

That means, that the function f will be drawn on the interval a to b. Try this now

```
> plot(f,-12..12);
```

An example of a function in two variables

```
> g:=(x,y)->(x^2-y^2)/(x^2+y^2);
                                   2    2
                                  x  - y
                    g := (x,y) -> -------
                                   2    2
                                  x  + y

> g(1,2);
                              -3/5

> g(1,x);
```

```
      2
1 - x
------
      2
1 + x
```

Functions of two variables can be graphed with the `plot3d` command

```
plot3d( f, a..b, c..d )
```

```
> plot3d(g, -1..1, -1..1);
```

The examples above are plots of functions. The other possibility is to graph an *expression*. The syntax and examples for plotting expressions is

```
plot( f(x), x=a..b )
plot3d( f(x,y), x=a..b, y=c..d )
```

```
> plot( sin(x)/x, x=-12..12 );
> plot3d( (x^2-y^2)/(x^2+y^2), x=-1..1, y=-1..1 );
```

13 Exercises

1. Calculate the first and second derivative of $\sin(x)\cos(x)$ wrt x.

2. Given the polynomial $y(x) = x^3 - 4x^2 + 4x - 1$ find the roots and any local minima and maxima of $y(x)$. Check your solutions with a plot of the polynomial.

3. Given $f = x^2 - 4$ calculate the integral of f and $\frac{1}{f}$ with respect to x. Check that Maples answers are correct by differentiating the results.

4. Compute the following integrals

$$\int_0^\infty e^{-t}\mathrm{d}t \qquad \text{and} \qquad \int_0^\infty e^{-t^2}\mathrm{d}t$$

5. Compute the following sums

$$\sum_{k=1}^{1000} k \qquad \text{and} \qquad \sum_{k=1}^{\infty} 1/k^2$$

6. Compute a formula for the sum of the first n integers and also the sum of the squares of the first n integers. I.e. compute and simplify the sums

$$\sum_{k=1}^{n} k \qquad \text{and} \qquad \sum_{k=1}^{n} k^2$$

7. Given the function $h(x) = 1 - x + sin(x)$, define the function in Maple and compute the value of the function h at $x = -2.0$ and then graph the function in the region $[-5,5]$.

8. Input the following Matrix A into Maple

$$A = \begin{pmatrix} a & 0 & 5 \\ 1 & 1 & 1 \\ -a & 0 & 0 \end{pmatrix}$$

   (a) Compute the characteristic polynomial of A. Hint: use the `charmat` and `det` commands in the linear algebra package `linalg`.

   (b) Compute the eigenvalues of A. Hint: use the `solve` or `factor` commands to find the roots of the characteristic polynomial.

9. Use the `solve` command to solve the linear system

$$\begin{vmatrix} 4x - 5y = 11 \\ 2x + y = 9 \end{vmatrix}$$

Maple: An Introduction

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March 12, 1992

1 Introduction

This introductory article contains basic information for the new user of Maple who wants to spend only a couple of hours learning Maple. This document covers how to use Maple as a calculator, recovering from errors, Maples notation for arithmetic, mathematical constants and functions, how to get help, how to solve equations, do linear algebra, and to define and plot mathematical functions in two and three dimensions. For each section there are several examples. At the end there are some exercises which you can try to solve. Note, we have not attempted to show any of Maples programming facilities in this article.

2 Getting Started

You should be able to start Maple by typing `maple`. The Maple logo should appear almost immediately. After the Maple prompt `>` has appeared, Maple is ready to receive your input commands. You should see

```
    |\^/|      MAPLE V
._|\|   |/|_. Copyright (c) 1981-1990 by the University of Waterloo.
 \  MAPLE  /   All rights reserved.  MAPLE is a registered trademark of
 <____ ____>   Waterloo Maple Software.
      |        Type ? for help.
>
```

Note, on workstations running X Windows, you can run a version of Maple which has a more sophisticated user interface called `xmaple`. Just type `xmaple` instead of `maple`. After a few seconds the Maple session window should appear which contains the Maple logo followed by the Maple prompt. You can now enter a command.

ATTENTION : every Maple command must end with a semicolon ;

This is a grammatical requirement of the Maple language. The semicolon tells Maple that this is the end of the input. Type now $\boxed{\texttt{4/6;}}$ followed by a return. Maple answers with

```
2/3
```

To refer to the latest result, one uses the double quote character ". Try it now

```
> ";
                        2/3
```

You can use the value of " in an expression as if it were a variable, for example

```
> 2*"+1;
                        7/3
```

In order to save results, one assigns them to a variable, e.g.

```
> R := ";
                      R := 7/3
```

If you want to interrupt a calculation push the interrupt key once. This is <Ctrl> C under Unix. When the prompt character appears, Maple is ready for further input. To exit Maple, you can type `quit`. Note, under `xmaple` you can click on the $\boxed{\text{Interrupt}}$ button to interrupt Maple and on the $\boxed{\text{Quit}}$ button to exit Maple.

3 Arithmetic and Maple Notation

The basic arithmetic operators and constants known to Maple are

Maple notation	Meaning	Mathematical notation
`x+y`	addition	$+$
`-x` and `x-y`	negation and subtraction	$-x$ and $x-y$
`x*y`	multiplication	xy
`x/y`	division	$\frac{x}{y}$
`x^y` or `x**y`	exponentiation	x^y
`k!`	factorial	$k!$
`I` or `sqrt(-1)`	complex unit	i or $\sqrt{-1}$
`Pi`		π
`infinity`		∞
`abs(x)`	absolute value	$\|x\|$
`sqrt(x)` or `x^(1/2)`	square root	$\sqrt{x}$
`exp(x)`	exponential function	e^x
`ln(x)` or `log(x)`	natural logarithm	$\ln x$
`sin(x)`	sine function	$\sin x$
`cos(x)`	cosine function	$\cos x$
`tan(x)`	tangent function	$\tan x$

Notice that Maple does arithmetic with integers exactly. I.e. exact arithmetic is used instead of decimal arithmetic. Use decimal numbers if you want decimal numbers. Examples

```
> 2*3+2/7;
                                    44/7

> 2.0*3.0+2.0/7;
                                 6.285714286
```

This principle works for formulae too. Use the `evalf` function if you want a decimal approximation.

```
>  sin(Pi/3);
                                        1/2
                                   1/2 3

>  evalf(");
                                 .8660254040
```

There is no limit on the length of integers in Maple. It is quite common to compute with integers several hundred digits long. It is also possible to do decimal arithmetic to more than the default 10 digits of precision. You can compute π to a 1000 digits if you want. This is done by assigning the the global variable Digits to the desired precision. Note, don't forget to reset Digits to 10 if you don't need more than 10 Digits! High precision operations take longer! Here are some examples

```
> 2^100;
                       1267650600228229401496703205376

> Digits := 50:
> evalf( sin(Pi/3) );

              .86602540378443864676372317075293618347140262690520

> Digits := 10:
```

In the above examples we have used the colon to terminate a command. Use the colon : instead of a semicolon ; if you don't want to see the output.

4 Mistakes and Errors

If you forget the semicolon don't panic! Maple will simply print another prompt indicating that it is still waiting for more input. Just type the semicolon and return and Maple will go ahead and compute the result. Note, this means that you can enter large expressions over one or more lines. Try to input a large expression like the following

```
> f := 4*x^4+3*x^3*y+2*x^2*y^2+
>      3*x*y^3+4*y^4;

                         4      3        2  2        3      4
                 f := 4 x  + 3 x  y + 2 x  y  + 3 x y  + 4 y
```

If you incorrectly input an expression, Maple will respond with a *syntax error*. A common error is to forget the semicolon and then to try to correct the problem by retyping the input. For example, one user typed

```
> 2*x+1
> 2*x+1
```

and Maple replied

```
syntax error:
2*x+1
^
```

The user forgot the semicolon after typing `2*x+1` and return. The user then typed in another `2*x+1`. Maple now says that there is an error and points to where it thinks the error is with the ^ character. What the user typed is equivalent to typing

```
> 2*x+1 2*x+1
```

Now you can see why Maple thinks the expression is wrong. To recover from a syntax error, just type a semicolon to clear the remaining input, and then reenter the command[1].

[1] In more sophisticated user interfaces, such as `xmaple`, you can edit your input

5 On-Line Help

If you need help for a Maple function, you can use the ? command. A ? by itself gives you general information. Try it now. E.g. ?min gives help on the min function.

```
FUNCTION: max - maximum of numbers
FUNCTION: min - minimum of numbers

CALLING SEQUENCE:
   max(x_1, x_2, ...)
   min( x_1, x_2, ...)

PARAMETERS:
   x_1, x_2, ... - any expressions

SYNOPSIS:
- The functions max and min return the maximum or minimum respectively of one
  or more arguments.

- Most often the arguments will be of type numeric, that is, integers, ration-
  als, or floats.  However, the functionality is more general, allowing any
  type of arguments for which an unevaluated function call may be returned.

EXAMPLES:
> max(3/2, 1.49);
                                    3/2
> min(3/2, 1.49);
                                    1.49
> max(3/5, evalf(ln(2)), 9/13);
                                .6931471806
> max(x+1, x+2, y);
                               max(x + 2, y)

SEE ALSO:  maximize, minimize
```

6 Simplification Commands

The output from Maple is not always in the simplest form. There are a number of commands such as collect, combine, expand, factor and simplify which can be used to simplify or rearrange the output into the desired form. For example, given the polynomial

```
> (x+y)*(x-y)-x^2;
                                          2
                           (x + y) (x - y) - x
```

if we multiply it out with the `expand` command the result is simpler

```
> expand(");
                                        2
                                     - y
```

Often factoring a polynomial yields a simpler result, e.g.

```
> x^4+x^2*y+2*x^2+2*x^3+2*x*y+2*x+y+1;

             4    2        2      3
            x  + x  y + 2 x  + 2 x  + 2 x y + 2 x + y + 1

> factor(");
                                     2   2
                              (x + 1)  (x  + y + 1)
```

The `simplify` command can simplify general expressions, e.g.

```
> simplify(exp(a+ln(b*exp(c))));

                                 b exp(a + c)
```

In this example the simplify command reduces a rational function

```
> (x^3-y^3)/(x^2+x*y+y^2);
                                      3    3
                                     x  - y
                                  -------------
                                   2          2
                                  x  + x y + y

> simplify(");
                                      x - y
```

7 Calculus Commands

Experiment with the integration, differentiation and summation commands. The Maple syntax for these functions and examples is as follows.

Maple notation	Meaning	Mathematical notation
`diff(f(x),x);`	Partial derivative	$\frac{d}{dx}f(x)$
`int(f(x),x);`	Indefinite integration	$\int f(x)dx$
`sum(f(x),x);`	Indefinite summation	$\sum_n f(n)$
`int(f(x),x=a..b);`	Definite integration	$\int_a^b f(x)dx$
`sum(f(k),k=a..b);`	Definite summation	$\sum_{k=a}^b f(k)$

```
> diff(ln(x),x);
                                       1/x

> diff(arcsin(a*x),x);
```

```
                      a
               --------------
                     2  2 1/2
               (1 - a  x )
```

```
> int(x*ln(x),x);
                          2               2
                     1/2 x  ln(x) - 1/4 x

> int(sqrt(1-x^2),x);
                                   2 1/2
                     1/2 x (1 - x )     + 1/2 arcsin(x)

> int(ln(x),x=1..2);
                              - 1 + 2 ln(2)

> sum(n^2,n);
                              3         2
                         1/3 n  - 1/2 n  + 1/6 n

> sum(a^k,k=1..n);
                              (n + 1)
                             a             a
                             -------- - -----
                              a - 1     a - 1
```

In the case of indefinite integration, you can test whether Maple's answer is right by differentiating the integral and subtracting in from the integrand. The difference should be zero! But Maple may not recognize that the difference is zero immediately. You may have to help Maple simplify the difference to 0. You can make use of `expand` and `simplify`. Example:

```
> f := (x^3+2*x^2-x)/(x^3-x^2+x-1);
                                     3      2
                                    x  + 2 x  - x
                              f := ---------------
                                     3    2
                                    x  - x  + x - 1

> int(f,x);
                                                  2
                        x + ln(x - 1) + ln(x  + 1)

> diff(",x);
                                  1           x
                            1 + ----- + 2 ------
                                x - 1       2
                                           x  + 1

> simplify("-f);
```

```
                                  0
```

8 The Solve Commands

You can solve single equations and systems of linear and non-linear equations exactly with the `solve` command. For approximate solutions, use the `fsolve` command. For ordinary differential equations use the `dsolve` command. And for recurrence equations use the `rsolve` command. These commands have the following syntax.

`solve(e, x)`	solve the equation e for the unknown x
`solve({e1, ..., en}, {x1, ..., xm})`	solve the equations $e_1, \ldots, e_n$ for $x_1, \ldots, x_m$
`fsolve(e, x)`	solve the equation e numerically for x
`fsolve({e1, ..., en}, {x1, ..., xm})`	solve numerically $e_1, \ldots, e_n$ for $x_1, ..., x_m$
`dsolve(e, y(x))`	solve the ODE e for $y(x)$
`dsolve({e, i1, ..., in}, y(x))`	solve the ODE e and initial conditions $i_1, ..., i_n$ for $y(x)$
`rsolve(e, f(n))`	solve the recurrence equation e for $f(n)$
`rsolve({e, i1, ..., in}, f(n))`	solve the recurrence e given initial conditions $i_1, ..., i_n$ for $f(n)$

For example, to solve the equation $x^3 - 6x = 5$ for x

```
> solve( x^3-6*x=5, x );
                                         1/2              1/2
                         -1, 1/2 + 1/2 21    , 1/2 - 1/2 21
```

To solve the ODE $y(x) + y''(x) = e^x$ given initial conditions $y(0) = 1$ and $y'(0) = 0$

```
> dsolve({y(x)+diff(y(x),x$2)=exp(x), y(0)=1, D(y)(0)=0}, y(x));

                 y(x) = 1/2 exp(x) - 1/2 sin(x) + 1/2 cos(x)
```

9 Other Commands and Functions

Maple knows about the elementary functions `ln, exp, sin, cos, tan,` etc. Some other special functions that you might need are

Maple notation	Function	Mathematical notation
`erf(x)`	Error function	$erf(x)$
`binomial(n,k)`	Binomial function	$\binom{n}{k}$
`GAMMA(x)`	Gamma function	$\Gamma(x)$
`Psi(x)`	Poly Gamma function	$\psi(x)$
`Zeta(x)`	Riemann Zeta function	$\zeta(x)$
`BesselJ(v,x)`	Bessel function of the first kind	$J_\nu(x)$

See also ?integer for a list of functions for integers and ?polynom for a list of commands for computing with polynomials. See ?mod for a list of commands for computing over finite rings and fields.

10 Data Structures

Maple uses sequences, lists, sets, tables and arrays for representing more complicated data. See ?sequences, ?list, ?set, ?table, ?array for more detailed help and examples. A sequence is a sequence of expressions separated by commas. The seq function is a very useful function for creating sequences. It's syntax is

```
seq( f(i), i=a..b )
```

Lists group sequences and are created using square brackets [,]. Sets are like lists except duplicate entries are removed. Sets are created using squiggley brackets {, }. Examples

```
> S1 := 1,5,3;
                          S1 := 1, 5, 3

> max(S1);
                                5

> S2 := seq(i^2, i=1..5);
                       S2 := 1, 4, 9, 16, 25

> S2[3];
                                9

> S1 := {x,y,z,y};
                         S1 := {x, y, z}

> S1 union {w,x};
                          {x, y, z, w}

> L1 := [x,y,z,y];
                        L1 := [x, y, z, y]

> L2 := [L1[4],L1[2..3]];
                         L2 := [y, y, z]
```

11 The Linear Algebra Package

In Maple a vector is represented as a one-dimensional array, and a matrix is represented as a two-dimensional array. See `?vector` and `?matrix` for detailed help on vectors and matrices.

In Maple there are many packages for special applications. The `linalg` package contains many functions from linear algebra for computing with vectors and matrices. In order to use a package, you must load the package using the `with` command, e.g.

```
> with(linalg);
Warning: new definition for   norm
Warning: new definition for   trace
[BlockDiagonal, GramSchmidt, JordanBlock, add, addcol, addrow, adj, adjoint,
    angle, augment, backsub, band, basis, bezout, charmat, charpoly, col,
    coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod,
    curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals,
    eigenvects, equal, exponential, extend, ffgausselim, fibonacci, frobenius,
    gausselim, gaussjord, genmatrix, grad, hadamard, hermite, hessian,
    hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse,
    ismith, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve,
    matrix, minor, minpoly, mulcol, mulrow, multiply, norm, nullspace, orthog,
    permanent, pivot, potential, randmatrix, range, rank, row, rowdim,
    rowspace, rowspan, rref, scalarmul, singularvals, smith, stack, submatrix,
    subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace,
    transpose, vandermonde, vecpotent, vectdim, vector]
```

You can now use any of the functions listed. The `matrix` command can be used to input a matrix. In the next example the inverse and determinant of a 3 by 3 matrix is computed.

```
> A := matrix([[x-1,2,3],[0,x-2,2],[2,1,x-3]]);
```

$$A := \begin{pmatrix} x-1 & 2 & 3 \\ 0 & x-2 & 2 \\ 2 & 1 & x-3 \end{pmatrix}$$

```
> det(A);
```

$$x^3 - 6x^2 + 3x + 16$$

```
> inverse(A);
```

$$\begin{pmatrix} \frac{x^2-5x+4}{x^3-6x^2+3x+16} & -\frac{2x-9}{x^3-6x^2+3x+16} & -\frac{3x-10}{x^3-6x^2+3x+16} \\ \frac{4}{x^3-6x^2+3x+16} & \frac{x^2-4x-3}{x^3-6x^2+3x+16} & -\frac{2x-2}{x^3-6x^2+3x+16} \\ -\frac{2x-4}{x^3-6x^2+3x+16} & -\frac{x-5}{x^3-6x^2+3x+16} & \frac{(x-1)(x-2)}{x^3-6x^2+3x+16} \end{pmatrix}$$

If you type `?packages` you will get a list of all the known packages to Maple and what they are. In particular, this includes

Package	Explanation
`numtheory`	number theoretic functions
`combinat`	combinatorial functions
`stats`	statistical functions
`geometry`	functions for dealing with two dimensional euclidean geometry
`orthopoly`	definition of orthogonal polynomials
`simplex`	functions for linear optimization problems

12 Functions and Graphics

Mathematical functions of one or more variables can be defined in Maple. The function can be evaluated at either numerical or symbolic expressions. For example

```
> f:=x->sin(x)/x;
                              sin(x)
                    f := x -> ------
                                 x

> f(2.0);
                       .4546487134

> f(t);
                          sin(t)
                          ------
                             t
```

Functions can be graphed with the `plot` command which has the following syntax.

```
plot( f, a..b )
```

That means, that the function f will be drawn on the interval a to b. Try this now

```
> plot(f,-12..12);
```

An example of a function in two variables

```
> g:=(x,y)->(x^2-y^2)/(x^2+y^2);
                                2    2
                               x  - y
                 g := (x,y) -> -------
                                2    2
                               x  + y

> g(1,2);
                          -3/5

> g(1,x);
```

```
     2
1 - x
------
     2
1 + x
```

Functions of two variables can be graphed with the `plot3d` command

```
plot3d( f, a..b, c..d )
```

```
> plot3d(g, -1..1, -1..1);
```

The examples above are plots of functions. The other possibility is to graph an *expression*. The syntax and examples for plotting expressions is

```
plot( f(x), x=a..b )
plot3d( f(x,y), x=a..b, y=c..d )
```

```
> plot( sin(x)/x, x=-12..12 );
> plot3d( (x^2-y^2)/(x^2+y^2), x=-1..1, y=-1..1 );
```

13 Exercises

1. Calculate the first and second derivative of $\sin(x)\cos(x)$ wrt x.

2. Given the polynomial $y(x) = x^3 - 4x^2 + 4x - 1$ find the roots and any local minima and maxima of $y(x)$. Check your solutions with a plot of the polynomial.

3. Given $f = x^2 - 4$ calculate the integral of f and $\frac{1}{f}$ with respect to x. Check that Maples answers are correct by differentiating the results.

4. Compute the following integrals

$$\int_0^\infty e^{-t}\mathrm{d}t \qquad \text{and} \qquad \int_0^\infty e^{-t^2}\mathrm{d}t$$

5. Compute the following sums

$$\sum_{k=1}^{1000} k \qquad \text{and} \qquad \sum_{k=1}^{\infty} 1/k^2$$

6. Compute a formula for the sum of the first n integers and also the sum of the squares of the first n integers. I.e. compute and simplify the sums

$$\sum_{k=1}^{n} k \qquad \text{and} \qquad \sum_{k=1}^{n} k^2$$

7. Given the function $h(x) = 1 - x + sin(x)$, define the function in Maple and compute the value of the function h at $x = -2.0$ and then graph the function in the region $[-5,5]$.

8. Input the following Matrix A into Maple

$$A = \begin{pmatrix} a & 0 & 5 \\ 1 & 1 & 1 \\ -a & 0 & 0 \end{pmatrix}$$

   (a) Compute the characteristic polynomial of A. Hint: use the `charmat` and `det` commands in the linear algebra package `linalg`.

   (b) Compute the eigenvalues of A. Hint: use the `solve` or `factor` commands to find the roots of the characteristic polynomial.

9. Use the `solve` command to solve the linear system

$$\left| \begin{array}{r} 4x - 5y = 11 \\ 2x + y = 9 \end{array} \right|$$

Maple: An Introduction

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March 12, 1992

1 Introduction

This introductory article contains basic information for the new user of Maple who wants to spend only a couple of hours learning Maple. This document covers how to use Maple as a calculator, recovering from errors, Maples notation for arithmetic, mathematical constants and functions, how to get help, how to solve equations, do linear algebra, and to define and plot mathematical functions in two and three dimensions. For each section there are several examples. At the end there are some exercises which you can try to solve. Note, we have not attempted to show any of Maples programming facilities in this article.

2 Getting Started

You should be able to start Maple by typing `maple`. The Maple logo should appear almost immediately. After the Maple prompt `>` has appeared, Maple is ready to receive your input commands. You should see

```
    |\^/|      MAPLE V
._|\|   |/|_.  Copyright (c) 1981-1990 by the University of Waterloo.
 \  MAPLE  /   All rights reserved.  MAPLE is a registered trademark of
 <____ ____>   Waterloo Maple Software.
      |        Type ? for help.
>
```

Note, on workstations running X Windows, you can run a version of Maple which has a more sophisticated user interface called `xmaple`. Just type `xmaple` instead of `maple`. After a few seconds the Maple session window should appear which contains the Maple logo followed by the Maple prompt. You can now enter a command.

ATTENTION : every Maple command must end with a semicolon ;

This is a grammatical requirement of the Maple language. The semicolon tells Maple that this is the end of the input. Type now $\boxed{\texttt{4/6;}}$ followed by a return. Maple answers with

```
2/3
```

To refer to the latest result, one uses the double quote character ". Try it now

```
> ";
                                2/3
```

You can use the value of " in an expression as if it were a variable, for example

```
> 2*"+1;
                                7/3
```

In order to save results, one assigns them to a variable, e.g.

```
> R := ";
                              R := 7/3
```

If you want to interrupt a calculation push the interrupt key once. This is <Ctrl> C under Unix. When the prompt character appears, Maple is ready for further input. To exit Maple, you can type `quit`. Note, under `xmaple` you can click on the $\boxed{\text{Interrupt}}$ button to interrupt Maple and on the $\boxed{\text{Quit}}$ button to exit Maple.

3 Arithmetic and Maple Notation

The basic arithmetic operators and constants known to Maple are

Maple notation	Meaning	Mathematical notation
`x+y`	addition	$+$
`-x` and `x-y`	negation and subtraction	$-x$ and $x - y$
`x*y`	multiplication	xy
`x/y`	division	$\frac{x}{y}$
`x^y` or `x**y`	exponentiation	x^y
`k!`	factorial	$k!$
`I` or `sqrt(-1)`	complex unit	i or $\sqrt{-1}$
`Pi`		π
`infinity`		∞
`abs(x)`	absolute value	$\|x\|$
`sqrt(x)` or `x^(1/2)`	square root	$\sqrt{x}$
`exp(x)`	exponential function	e^x
`ln(x)` or `log(x)`	natural logarithm	$\ln x$
`sin(x)`	sine function	$\sin x$
`cos(x)`	cosine function	$\cos x$
`tan(x)`	tangent function	$\tan x$

Notice that Maple does arithmetic with integers exactly. I.e. exact arithmetic is used instead of decimal arithmetic. Use decimal numbers if you want decimal numbers. Examples

```
> 2*3+2/7;
                                   44/7

> 2.0*3.0+2.0/7;
                                6.285714286
```

This principle works for formulae too. Use the `evalf` function if you want a decimal approximation.

```
>  sin(Pi/3);
                                      1/2
                                 1/2 3

>  evalf(");
                                .8660254040
```

There is no limit on the length of integers in Maple. It is quite common to compute with integers several hundred digits long. It is also possible to do decimal arithmetic to more than the default 10 digits of precision. You can compute π to a 1000 digits if you want. This is done by assigning the the global variable Digits to the desired precision. Note, don't forget to reset Digits to 10 if you don't need more than 10 Digits! High precision operations take longer! Here are some examples

```
> 2^100;
                      1267650600228229401496703205376

> Digits := 50:
> evalf( sin(Pi/3) );

           .86602540378443864676372317075293618347140262690520

> Digits := 10:
```

In the above examples we have used the colon to terminate a command. Use the colon : instead of a semicolon ; if you don't want to see the output.

4 Mistakes and Errors

If you forget the semicolon don't panic! Maple will simply print another prompt indicating that it is still waiting for more input. Just type the semicolon and return and Maple will go ahead and compute the result. Note, this means that you can enter large expressions over one or more lines. Try to input a large expression like the following

```
> f := 4*x^4+3*x^3*y+2*x^2*y^2+
>      3*x*y^3+4*y^4;

                        4      3        2  2        3      4
              f := 4 x  + 3 x  y + 2 x  y  + 3 x y  + 4 y
```

If you incorrectly input an expression, Maple will respond with a *syntax error*. A common error is to forget the semicolon and then to try to correct the problem by retyping the input. For example, one user typed

```
> 2*x+1
> 2*x+1
```

and Maple replied

```
syntax error:
2*x+1
^
```

The user forgot the semicolon after typing `2*x+1` and return. The user then typed in another `2*x+1`. Maple now says that there is an error and points to where it thinks the error is with the ^ character. What the user typed is equivalent to typing

```
> 2*x+1 2*x+1
```

Now you can see why Maple thinks the expression is wrong. To recover from a syntax error, just type a semicolon to clear the remaining input, and then reenter the command[1].

[1] In more sophisticated user interfaces, such as `xmaple`, you can edit your input

5 On-Line Help

If you need help for a Maple function, you can use the `?` command. A `?` by itself gives you general information. Try it now. E.g. `?min` gives help on the `min` function.

```
FUNCTION: max - maximum of numbers
FUNCTION: min - minimum of numbers

CALLING SEQUENCE:
   max(x_1, x_2, ...)
   min( x_1, x_2, ...)

PARAMETERS:
   x_1, x_2, ... - any expressions

SYNOPSIS:
- The functions max and min return the maximum or minimum respectively of one
  or more arguments.

- Most often the arguments will be of type numeric, that is, integers, ration-
  als, or floats.  However, the functionality is more general, allowing any
  type of arguments for which an unevaluated function call may be returned.

EXAMPLES:
> max(3/2, 1.49);
                                    3/2
> min(3/2, 1.49);
                                    1.49
> max(3/5, evalf(ln(2)), 9/13);
                                .6931471806
> max(x+1, x+2, y);
                               max(x + 2, y)

SEE ALSO:  maximize, minimize
```

6 Simplification Commands

The output from Maple is not always in the simplest form. There are a number of commands such as `collect`, `combine`, `expand`, `factor` and `simplify` which can be used to simplify or rearrange the output into the desired form. For example, given the polynomial

```
> (x+y)*(x-y)-x^2;
                                                  2
                              (x + y) (x - y) - x
```

if we multiply it out with the `expand` command the result is simpler

```
> expand(");
                                        2
                                     - y
```

Often factoring a polynomial yields a simpler result, e.g.

```
> x^4+x^2*y+2*x^2+2*x^3+2*x*y+2*x+y+1;

                 4    2         2      3
                x  + x  y + 2 x  + 2 x  + 2 x y + 2 x + y + 1

> factor(");
                                    2   2
                             (x + 1)  (x  + y + 1)
```

The `simplify` command can simplify general expressions, e.g.

```
> simplify(exp(a+ln(b*exp(c))));

                                 b exp(a + c)
```

In this example the simplify command reduces a rational function

```
> (x^3-y^3)/(x^2+x*y+y^2);
                                     3    3
                                    x  - y
                                 -------------
                                  2          2
                                 x  + x y + y

> simplify(");
                                     x - y
```

7 Calculus Commands

Experiment with the integration, differentiation and summation commands. The Maple syntax for these functions and examples is as follows.

Maple notation	Meaning	Mathematical notation
`diff(f(x),x);`	Partial derivative	$\frac{d}{dx}f(x)$
`int(f(x),x);`	Indefinite integration	$\int f(x)dx$
`sum(f(x),x);`	Indefinite summation	$\sum_n f(n)$
`int(f(x),x=a..b);`	Definite integration	$\int_a^b f(x)dx$
`sum(f(k),k=a..b);`	Definite summation	$\sum_{k=a}^b f(k)$

```
> diff(ln(x),x);
                                      1/x

> diff(arcsin(a*x),x);
```

```
                                   a
                             --------------
                                   2  2 1/2
                             (1 - a  x )

> int(x*ln(x),x);
                              2              2
                         1/2 x  ln(x) - 1/4 x

> int(sqrt(1-x^2),x);
                                 2 1/2
                   1/2 x (1 - x )    + 1/2 arcsin(x)

> int(ln(x),x=1..2);
                             - 1 + 2 ln(2)

> sum(n^2,n);
                              3        2
                         1/3 n  - 1/2 n  + 1/6 n

> sum(a^k,k=1..n);
                             (n + 1)
                            a           a
                            -------- - -----
                             a - 1     a - 1
```

In the case of indefinite integration, you can test whether Maple's answer is right by differentiating the integral and subtracting in from the integrand. The difference should be zero! But Maple may not recognize that the difference is zero immediately. You may have to help Maple simplify the difference to 0. You can make use of `expand` and `simplify`. Example:

```
> f := (x^3+2*x^2-x)/(x^3-x^2+x-1);
                                  3      2
                                 x  + 2 x  - x
                          f := ---------------
                                 3    2
                                x  - x  + x - 1

> int(f,x);
                                                2
                       x + ln(x - 1) + ln(x  + 1)

> diff(",x);
                                 1          x
                          1 + ----- + 2 ------
                              x - 1       2
                                         x  + 1

> simplify("-f);
```

0

8 The Solve Commands

You can solve single equations and systems of linear and non-linear equations exactly with the `solve` command. For approximate solutions, use the `fsolve` command. For ordinary differential equations use the `dsolve` command. And for recurrence equations use the `rsolve` command. These commands have the following syntax.

`solve(e, x)`	solve the equation e for the unknown x
`solve({e1, ..., en}, {x1, ..., xm})`	solve the equations $e_1, \ldots, e_n$ for $x_1, \ldots, x_m$
`fsolve(e, x)`	solve the equation e numerically for x
`fsolve({e1, ..., en}, {x1, ..., xm})`	solve numerically $e_1, \ldots, e_n$ for $x_1, ..., x_m$
`dsolve(e, y(x))`	solve the ODE e for $y(x)$
`dsolve({e, i1, ..., in}, y(x))`	solve the ODE e and initial conditions $i_1, ..., i_n$ for $y(x)$
`rsolve(e, f(n))`	solve the recurrence equation e for $f(n)$
`rsolve({e, i1, ..., in}, f(n))`	solve the recurrence e given initial conditions $i_1, ..., i_n$ for $f(n)$

For example, to solve the equation $x^3 - 6x = 5$ for x

```
> solve( x^3-6*x=5, x );
                                    1/2              1/2
                    -1, 1/2 + 1/2 21   , 1/2 - 1/2 21
```

To solve the ODE $y(x) + y''(x) = e^x$ given initial conditions $y(0) = 1$ and $y'(0) = 0$

```
> dsolve({y(x)+diff(y(x),x$2)=exp(x), y(0)=1, D(y)(0)=0}, y(x));

                y(x) = 1/2 exp(x) - 1/2 sin(x) + 1/2 cos(x)
```

9 Other Commands and Functions

Maple knows about the elementary functions `ln, exp, sin, cos, tan,` etc. Some other special functions that you might need are

Maple notation	Function	Mathematical notation
`erf(x)`	Error function	$erf(x)$
`binomial(n,k)`	Binomial function	$\binom{n}{k}$
`GAMMA(x)`	Gamma function	$\Gamma(x)$
`Psi(x)`	Poly Gamma function	$\psi(x)$
`Zeta(x)`	Riemann Zeta function	$\zeta(x)$
`BesselJ(v,x)`	Bessel function of the first kind	$J_\nu(x)$

See also `?integer` for a list of functions for integers and `?polynom` for a list of commands for computing with polynomials. See `?mod` for a list of commands for computing over finite rings and fields.

10 Data Structures

Maple uses sequences, lists, sets, tables and arrays for representing more complicated data. See `?sequences`, `?list`, `?set`, `?table`, `?array` for more detailed help and examples. A sequence is a sequence of expressions separated by commas. The `seq` function is a very useful function for creating sequences. It's syntax is

```
seq( f(i), i=a..b )
```

Lists group sequences and are created using square brackets `[, ]`. Sets are like lists except duplicate entries are removed. Sets are created using squiggley brackets {, }. Examples

```
> S1 := 1,5,3;
                              S1 := 1, 5, 3

> max(S1);
                                    5

> S2 := seq(i^2, i=1..5);
                          S2 := 1, 4, 9, 16, 25

> S2[3];
                                    9

> S1 := {x,y,z,y};
                             S1 := {x, y, z}

> S1 union {w,x};
                              {x, y, z, w}

> L1 := [x,y,z,y];
                            L1 := [x, y, z, y]

> L2 := [L1[4],L1[2..3]];
                             L2 := [y, y, z]
```

11 The Linear Algebra Package

In Maple a vector is represented as a one-dimensional array, and a matrix is represented as a two-dimensional array. See `?vector` and `?matrix` for detailed help on vectors and matrices.

In Maple there are many packages for special applications. The `linalg` package contains many functions from linear algebra for computing with vectors and matrices. In order to use a package, you must load the package using the `with` command, e.g.

```
> with(linalg);
Warning: new definition for   norm
Warning: new definition for   trace
[BlockDiagonal, GramSchmidt, JordanBlock, add, addcol, addrow, adj, adjoint,
    angle, augment, backsub, band, basis, bezout, charmat, charpoly, col,
    coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod,
    curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals,
    eigenvects, equal, exponential, extend, ffgausselim, fibonacci, frobenius,
    gausselim, gaussjord, genmatrix, grad, hadamard, hermite, hessian,
    hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse,
    ismith, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve,
    matrix, minor, minpoly, mulcol, mulrow, multiply, norm, nullspace, orthog,
    permanent, pivot, potential, randmatrix, range, rank, row, rowdim,
    rowspace, rowspan, rref, scalarmul, singularvals, smith, stack, submatrix,
    subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace,
    transpose, vandermonde, vecpotent, vectdim, vector]
```

You can now use any of the functions listed. The `matrix` command can be used to input a matrix. In the next example the inverse and determinant of a 3 by 3 matrix is computed.

```
> A := matrix([[x-1,2,3],[0,x-2,2],[2,1,x-3]]);
```

$$A := \begin{pmatrix} x-1 & 2 & 3 \\ 0 & x-2 & 2 \\ 2 & 1 & x-3 \end{pmatrix}$$

```
> det(A);
```

$$x^3 - 6x^2 + 3x + 16$$

```
> inverse(A);
```

$$\begin{pmatrix} \frac{x^2-5x+4}{x^3-6x^2+3x+16} & -\frac{2x-9}{x^3-6x^2+3x+16} & -\frac{3x-10}{x^3-6x^2+3x+16} \\ \frac{4}{x^3-6x^2+3x+16} & \frac{x^2-4x-3}{x^3-6x^2+3x+16} & -\frac{2x-2}{x^3-6x^2+3x+16} \\ -\frac{2x-4}{x^3-6x^2+3x+16} & -\frac{x-5}{x^3-6x^2+3x+16} & \frac{(x-1)(x-2)}{x^3-6x^2+3x+16} \end{pmatrix}$$

If you type `?packages` you will get a list of all the known packages to Maple and what they are. In particular, this includes

Package	Explanation
`numtheory`	number theoretic functions
`combinat`	combinatorial functions
`stats`	statistical functions
`geometry`	functions for dealing with two dimensional euclidean geometry
`orthopoly`	definition of orthogonal polynomials
`simplex`	functions for linear optimization problems

12 Functions and Graphics

Mathematical functions of one or more variables can be defined in Maple. The function can be evaluated at either numerical or symbolic expressions. For example

```
> f:=x->sin(x)/x;
                              sin(x)
                    f := x -> ------
                                 x

> f(2.0);
                       .4546487134

> f(t);
                          sin(t)
                          ------
                             t
```

Functions can be graphed with the `plot` command which has the following syntax.

```
plot( f, a..b )
```

That means, that the function f will be drawn on the interval a to b. Try this now

```
> plot(f,-12..12);
```

An example of a function in two variables

```
> g:=(x,y)->(x^2-y^2)/(x^2+y^2);
                                    2    2
                                   x  - y
                   g := (x,y) -> -------
                                    2    2
                                   x  + y

> g(1,2);
                           -3/5

> g(1,x);
```

```
     2
1 - x
------
     2
1 + x
```

Functions of two variables can be graphed with the `plot3d` command

```
plot3d( f, a..b, c..d )
```

```
> plot3d(g, -1..1, -1..1);
```

The examples above are plots of functions. The other possibility is to graph an *expression*. The syntax and examples for plotting expressions is

```
plot( f(x), x=a..b )
plot3d( f(x,y), x=a..b, y=c..d )
```

```
> plot( sin(x)/x, x=-12..12 );
> plot3d( (x^2-y^2)/(x^2+y^2), x=-1..1, y=-1..1 );
```

13 Exercises

1. Calculate the first and second derivative of $\sin(x)\cos(x)$ wrt x.

2. Given the polynomial $y(x) = x^3 - 4x^2 + 4x - 1$ find the roots and any local minima and maxima of $y(x)$. Check your solutions with a plot of the polynomial.

3. Given $f = x^2 - 4$ calculate the integral of f and $\frac{1}{f}$ with respect to x. Check that Maples answers are correct by differentiating the results.

4. Compute the following integrals

$$\int_0^\infty e^{-t}\mathrm{d}t \qquad \text{and} \qquad \int_0^\infty e^{-t^2}\mathrm{d}t$$

5. Compute the following sums

$$\sum_{k=1}^{1000} k \qquad \text{and} \qquad \sum_{k=1}^{\infty} 1/k^2$$

6. Compute a formula for the sum of the first n integers and also the sum of the squares of the first n integers. I.e. compute and simplify the sums

$$\sum_{k=1}^{n} k \qquad \text{and} \qquad \sum_{k=1}^{n} k^2$$

7. Given the function $h(x) = 1 - x + sin(x)$, define the function in Maple and compute the value of the function h at $x = -2.0$ and then graph the function in the region $[-5,5]$.

8. Input the following Matrix A into Maple

$$A = \begin{pmatrix} a & 0 & 5 \\ 1 & 1 & 1 \\ -a & 0 & 0 \end{pmatrix}$$

   (a) Compute the characteristic polynomial of A. Hint: use the `charmat` and `det` commands in the linear algebra package `linalg`.

   (b) Compute the eigenvalues of A. Hint: use the `solve` or `factor` commands to find the roots of the characteristic polynomial.

9. Use the `solve` command to solve the linear system

$$\begin{vmatrix} 4x - 5y = 11 \\ 2x + y = 9 \end{vmatrix}$$