EXAM 4
MAT220 Fall 2022
Form 470541

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1. The orthogonal projection of the vector $(4, 1, 3)$ on the plane generated by the vectors $(1, 3, -2)$ and $(-1, 2, 4)$ is,
   (A) $(17/85, 143/285)$
   (B) $(0, 5, 2)$
   (C) $(-43/285, 656/285, 87/48)$
   (D) $(-1, 2, 2)$
   (E) $(-92/285, 439/285, 94/57)$

2. Among all the functions of the form $g(x) = ax^2$ the one that best fits the four points $(-2, 2), (-1, 1), (1, 1), (2, 2)$ is given by:
   (A) $g(x) = \frac{1}{2}x^2$
   (B) $g(x) = x^2$
   (C) $g(x) = \frac{18}{37}x^2$
   (D) $g(x) = \frac{3}{5}x^2$
   (E) $g(x) = \frac{9}{17}x^2$

3. Let T be the linear transformation that takes a 2 by 2 matrix into another 2 by 2 matrix defined by:
$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$
   In the standard basis of 2 by 2 matrices $\{E_{11}, E_{12}, E_{21}, E_{22}\}$ (where E_{ij} denotes the 2 by 2 matrix with zeros everywhere except at position ij where it has a one), the matrix representation of T is given in sage notation (i.e. by row) by:
   (A) matrix(2,2,[1,-1,-1,1])
   (B) matrix(4,4,[0,0,0,1, 0,-1,0,0, 0,0,-1,0, 1,0,0,0])
   (C) matrix(4,4,[0,0,0,a, 0,-b,0,0, 0,0,-c,0, d,0,0,0])
   (D) matrix(4,4,[0,0,0,d, 0,-b,0,0, 0,0,-c,0, a,0,0,0])
   (E) matrix(2,2,[d,-b,-c,a])

4. Consider the linear operator $T : R^2 \to R^2$, defined by $T(x, y) = (-y, x)$. The matrix representation of the operator T in the standard basis of R^2 is given by:
   (A) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(E) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

5. The matrix that represents a reflection on the plane $2x - y + z = 0$, is given (in the standard bases) by:

(A) $\begin{bmatrix} -1 & 2 & -2 \\ 2 & 2 & 1 \\ -2 & 1 & 2 \end{bmatrix}$

(B) It can't be computed with the information given

(C) $\frac{1}{3}\begin{bmatrix} -1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

(D) $\frac{1}{3}\begin{bmatrix} -2 & 2 & -2 \\ 2 & 2 & 1 \\ -2 & 1 & 2 \end{bmatrix}$

(E) $\frac{1}{3}\begin{bmatrix} -1 & 2 & -2 \\ 2 & 2 & 1 \\ -2 & 1 & 2 \end{bmatrix}$

6. Take a vector $\boldsymbol{u}$ in R^3, multiply it by $\sqrt{2}$, then rotate it about the z-axis 45^o counterclockwise, and finally reflect it about the xz-plane. Call the resulting vector $\boldsymbol{v}$.

The matrix that represents the transformation $\boldsymbol{u} \mapsto \boldsymbol{v}$ is given, in the standard bases, by:

(A) $\begin{bmatrix} 1 & -1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix}$

(B) $\begin{bmatrix} \sqrt{2} & -\sqrt{2} & 0 \\ -\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix}$

(D) $\begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & -1 \end{bmatrix}$

(E) $\begin{bmatrix} 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \\ -1 & 0 & -1 \end{bmatrix}$

7. Let $T : P_2 \mapsto P_3$, where P_k is the linear space of polynomials of degree at most k. Define $T(p(x)) = xp(x)$. The coordinates, in the standard bases for polynomials, of the elements of the image space (i.e. range) of T are generated by:
   (A) $(0, x, 0, 0), (0, 0, x^2, 0), (0, 0, 0, x^3)$
   (B) Doesn't make sense, T is not a linear transformation
   (C) all of P_3 since the nullity of T is 0
   (D) $(1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 1, 0), (0, 0, 0, 1)$
   (E) $(0, 1, 0, 0), (0, 0, 1, 0), (0, 0, 0, 1)$

8. The determinant of A where,
$$A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & -4 \end{bmatrix}$$
   is closest to:
   (A) 1
   (B) 28
   (C) 42
   (D) 0
   (E) 24

9. Let f be a linear transformation from a vector space of dimension 3 into itself, and let $\{\sigma_1, \sigma_2, \sigma_3\}$ be the cannonical orthonormal basis in the space. If
$$\begin{aligned} f(\sigma_3 - \sigma_2) &= 0 \\ f(\sigma_1) &= \sigma_1 \\ f(\sigma_2 + \sigma_3) &= 2(\sigma_2 + \sigma_3) \end{aligned}$$
   Then,
   (A) f is orthogonal
   (B) f has determinant -2
   (C) f has σ_2 as an eigenvector
   (D) f is nonsingular

(E) f is symmetric

10. The following matrix A has $\lambda = 2$ and $\lambda = 8$ as its eigenvalues:

$$A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

Let P be the orthogonal matrix that diagonalizes A. In other words $A = PDP^T$. You can check that,

$$P = \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3} \\ 0 & \frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{3} \end{bmatrix}$$

Then the linear space of eigenvectors associated to the eigenvalue $\lambda = 2$ is generated by:

(A) the first two columns of the matrix P
(B) the first two rows of the matrix P
(C) the last two columns of the matrix P
(D) the last two rows of the matrix P
(E) the last column of the matrix P

11. Consider the transformation: Reflection on the plane orthogonal to $(2, -1, 2)$ followed by a rotation in 30 degrees ccw on the plane orthogonal to $(-2, 1, 2)$. The matrix of this transformation is given by:

(A)
$$\begin{bmatrix} 0.44 & 0.62 & -0.64 \\ 0.54 & 0.39 & 0.75 \\ -0.70 & 0.68 & 0.17 \end{bmatrix}$$

(B)
$$\begin{bmatrix} -0.15 & -11.68 & -1.03 \\ 0.10 & 8.88 & 53.78 \\ -0.97 & -76.55 & -50.23 \end{bmatrix}$$

(C)
$$\begin{bmatrix} 0.54 & -0.62 & 0.64 \\ 0.45 & 0.93 & 0.75 \\ 0.70 & -0.68 & 0.17 \end{bmatrix}$$

(D)
$$\begin{bmatrix} -0.15 & 0.17 & -0.96 \\ 0.10 & 0.98 & 0.16 \\ -0.97 & 0.08 & 0.17 \end{bmatrix}$$

(E) This does not make sense. The transformation is non linear.

12. Consider the $n \times n$ circular matrix $H = (h_{ij})$ with entries,

$$h_{ij} = \frac{1}{i+j-1}$$

It can be shown that the determinant of this matrix is always the reciprocal of an integer. In the case $n = 4$, the determinant of H is one over:

(A) 2667168

(B) 2160

(C) 6048000

(D) 1863134

(E) 604912

13. Consider the matrix,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Which of the following statements is false?

(A) $\lambda = 1$ is an eigenvalue of A

(B) A is singular

(C) A has three different eigenvalues

(D) $(0, -1, 1)$ is an eigenvector of A

(E) $(1, -1, 0)$ is an eigenvector of A

14. A checkboard matrix is a square matrix that alternates zeros and ones. i.e. $A = (a_{ij})$ with $a_{ij} = (1 + (-1)^{(i+j)})/2$. Which one of the following alternatives is NOT TRUE for a $7x7$ checkboard matrix?

(A) it is singular

(B) the nullity is 4

(C) the nullity is 5

(D) the rank is 2

(E) the columns are linearly dependent

15. Let T be the transformation of 2 by 2 real symmetric matrices defined by:

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix} \mapsto \begin{bmatrix} c & -b \\ -b & a \end{bmatrix}$$

Then wich of the following statements is **NOT** true?

(A) T is linear

(B) $\lambda = 2$ is an eigen value of T

- (C) $T^{-1} = T$
- (D) $\det(T) = -1$
- (E) The space of 2 by 2 real symmetric matrices with only zeros in the main diagonal is an eigenspace of T

16. The reflection of the vector $(1, 1/2, -1)$ on the plane $x - y - z = 0$ is given by:
    - (A) $(1, -1/2, 1)$
    - (B) $(0, 1/2, 0)$
    - (C) $(0, 3/2, 0)$
    - (D) $(0, -3/2, 0)$
    - (E) $(0, -1/2, 0)$

17. Let P denote the matrix representation, in the standard bases of R^3, of the orthogonal projection onto the plane $x + y + z = 0$ in R^3. Then, the inverse of P is,
    - (A) $2P - I$
    - (B) It Does not Exist.
    - (C) P^2
    - (D) $I - P$
    - (E) $(P + I)(P - I)$

18. Let E be the vector space of all the linear transformations from the euclidean plane to itself. Consider now the function T, that to each $f \in E$ assigns the vector $f(1, 1)$. Which one of the following alternatives is **NOT** true:
    - (A) The matrix representation of T is a 2 by 4 matrix
    - (B) T^{-1} does not exist
    - (C) T can be represented by a 2 by 2 matrix
    - (D) T is linear
    - (E) The range of T is the entire plane

19. Consider the following transformation of 3 dimensional euclidean space into itself: Rotation, on the xy plane, in 45 degrees ccw followed by orthogonal projection onto the plane $x - y = 0$. The kernel of this transformation is given by:
    - (A) The line through the origin in the direction of $(1, 1, 0)$
    - (B) The y axis
    - (C) The xz plane
    - (D) The transformation is invertible so its kernel is just $\{0\}$
    - (E) The plane $x = y$

20. Let

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 0 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

The eigenvalues of A^{10} are

(A) 2,0,1

(B) 1024, 0, 1

(C) 0,0,0

(D) 30, 20, 10

(E) 1000, 274, 1