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Theory of Events *

Ph. Blanchard[♭] and A. Jadczyk[♯†]

[♭] *Faculty of Physics and BiBoS, University of Bielefeld*
Universitätstr. 25, D-33615 Bielefeld

[♯] *Institute of Theoretical Physics, University of Wrocław*
Pl. Maxa Borna 9, PL-50 204 Wrocław

Abstract

We review what we call "event-enhanced formalism" of quantum theory. In this approach we explicitly assume classical nature of events. Given a quantum system, that is coupled to a classical one by a suitable coupling, classical events are being triggered. The trigerring process is partly random and partly deterministic. Within this new approach one can modelize real experimental events, including pointer readings of measuring devices. Our theory gives, for the first time, a unique algorithm that can be used for computer generation of experimental runs with individual quantum objects.

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† e-mail: ajad@ift.uni.wroc.pl

1 Introduction

We will talk about "theory of events". To be honest we should allow for the adjective "phenomenological". We will explain later our reasons for this restraint. This new theory enhances and extends the standard quantum formalism. It provides a solution to the quantum measurement problem. The usual formalism of quantum theory fails in this respect. Let us look, for instance, into a recent book on the subject, "The interpretation of quantum theory"[1]. There we can see both the difficulties as well as the methods that attempt to overcome them. We disagree with the optimism shared by many, perhaps by a majority of quantum physicists. They seem to believe that the problem is already solved, or almost solved. They use a magic spell, and at present the magic spell that is supposed to dissolve the problems is "decoherence". It is true that there are new ideas and new results in the decoherence approach. But these results did not quite solve the problem. Real–world–events, in particular pointer readings of measuring apparata, have never be obtained within this approach. Decoherence does not tell us yet how to programm a computer to simulate such events. A physicist, a human being, must intervene to decide what to decohere and how to decohere. Which basis is to be distinguished. What must be neglected and what must not? Which limit to take? That necessity of a human intervention is not a surprise. The standard quantum formalism simply has no resources that can be called for when we wish to *derive* the basic postulates about measurements and probabilities. These postulates are repeated in all textbooks. They are never *derived*. The usual probabilistic interpretation of quantum theory is *postulated* from outside. It is not deduced from within the formalism. That is rather unsatisfactory. We want to believe that quantum theory is fundamental, but its interpretation is so arbitray! Must it be so?

Many physicists would oppose. They disagree with such a criticism. They see that quantum theory is good, is excellent, because it gives excellent results. But there are other voices too. We like to recall John Bell's opinion

on this matter. He has studied the subject rather deeply. He emphasized it repeatedly [2, 3]: our problems with quantum measurements have a source. The reason is that the very concept of "measurement" *can not even be precisely defined* within the standard formalism. That is also our opinion. But not only we share his criticism. We also propose a way out that is new.
Our solution does not involve hidden variables (but we like to joke that the standard quantum state vector can be considered as a hidden variable). Our reasoning goes as follows:
First, we point out the reason *why* "measurement" could not be defined within the standard approach. It is true that the standard formalism of quantum theory has many sophisticated tools: it has Hilbert spaces, wave vectors, operators, spectral measures, POV measures. But it has no place for "events". What constitutes an event? The only candidate for an event that we can think of is change of a quantum state vector. But how do we observe state vectors? We can not see them directly. We were taught by Bohr and Heisenberg that any observation will disturb a quantum state. Well, unless the state is already known to us, then we can try to be clever and not to disturb it. But how can we know the state? We need a theory, that would help us to unswer these questions. We are proposing such a theory. We extend the standard formalism. We do it in a minimal way: just enough to accomodate classical events. We add explicitly a classical part to the quantum part, and we couple classical to the quantum. Then we define "experiments" and "measurements" within the so extended formalism. We can show that the standard postulates concerning measurements – in fact, in an enhanced and refined form – can be derived instead of being postulated.

This "event enhanced quantum theory", as we call it, gives experimental predictions that are stronger than those obtained from the standard theory. The new theory gives answers to more experimental questions than the old one. It provides algorithms for numerical simulations of experimental time series given by experiments with single quantum systems. In particular this new theory is falsifiable. But our programm is not yet complete. Our theory is based on an explicit selection of a classical subsystem. How to select what is classical? If we want to be on a save side as much as possible, or as long as possible, then we will shift “classical”into the observer's mind. But will we be save then? For how long? Soon we will need to extend our theory and to include a theory of mind and a theory of knowledge. That necessity will face us anyhow, pehaps even soon. But it is not clear that the cut must reside that far from the ordinary physics. For many practical applications the measuring apparatus itself, or its relevant part, can be considered classical. We need to derive such a splitting into classical and quantum from some clear principles. At present we do not know what these principles are, we can only guess.

At the present stage placement of the split is indeed phenomenological, and the coupling is phenomenological too. Both are simple to handle and easy to describe in our formalism. But where to put the Heisenberg's cut – that is arbitrary to some extent. Perhaps we need not worry too much? Perhaps relativity of the split is a new feature that will remain with us. We do not know. That is why we call our theory “phenomenological”. But we would like to stress that the standard, orthodox, pure quantum theory is not better in this respect. In fact, it is much worse. It is not even able to define what measurement is. It is not even a phenomenological theory. In fact, strictly speaking, it is not even a theory. It is partly an art, and that needs an artist. In this case it needs a physicist with his human experience and with his human intuition. Suppose we have a problem that needs quantum theory for its solution. Then our physicist, guided by his intuition, will replace the problem at hand by another problem, that can be handled. After that, guided by his experience, he will compute Green's function or whatsoever to get formulas out of this other problem. Finally, guided by his previous experience and by his intuition, he will interpret the formulas that he got, and he will predict some numbers for the experiment.
That job can not be left to a computing machine in an unmanned space–craft. We may feel proud that we are that necessary, that we can not be replaced

by machines. But would it not be better if we could spare our creativity for inventing new theories rather than spending it unnecessarily for application of the old ones?

Our theory is better in this respect. Once we have chosen a model – then reality, with all its events as they happen in time, can be simulated by a sufficiently powerful digital computer.

2 The formalism

Let us sketch the mathematical framework. To define events, we introduce a classical system C, and possible events will be identified with changes of a (pure) state of C. Let us consider the simplest situation corresponding to a finite set of possible events. If necessary, we can handle infinite dimensional generalizations of this framework. The space of states of the classical system, denoted by $\mathcal{S}_c$, has m states, labelled by $\alpha = 1, \ldots, m$. These are the pure states of C. They correspond to possible results of single observations of C. Statistical states of C are probability measures on $\mathcal{S}_c$ – in our case just sequences $p_\alpha \geq 0, \sum_\alpha p_\alpha = 1$. They describe ensambles of observations.
We will also need the algebra of (complex) observables of C. This will be the algebra $\mathcal{A}_c$ of complex functions on $\mathcal{S}_c$ – in our case just sequences $f_\alpha, \alpha = 1, \ldots, m$ of complex numbers.
It is convenient to use Hilbert space language even for the description of that simple classical system. Thus we introduce an m-dimensional Hilbert space $\mathcal{H}_c$ with a fixed basis, and we realize $\mathcal{A}_c$ as the algebra of diagonal matrices $F = \mathrm{diag}(f_1, \ldots, f_m)$.
Statistical states of C are then diagonal density matrices $\mathrm{diag}(p_1, \ldots, p_m)$, and pure states of C are vectors of the fixed basis of $\mathcal{H}_c$.
Events are ordered pairs of pure states $\alpha \to \beta$, $\alpha \neq \beta$. Each event can thus be represented by an $m \times m$ matrix with 1 at the (α, β) entry, zero otherwise. There are $m^2 - m$ possible events.

Statistical states are concerned with ensembles, while pure states and events concern individual systems.
The simplest classical system is a yes–no counter. It has only two distinct pure states. Its algebra of observables consists of 2×2 diagonal matrices.

We now come to the quantum system. Here we use the standard description.
Let Q be the quantum system whose bounded observables are from the algebra $\mathcal{A}_q$ of bounded operators on a Hilbert space $\mathcal{H}_q$. Its pure states are unit vectors in $\mathcal{H}_q$; proportional vectors describe the same quantum state. Statistical states of Q are given by non–negative density matrices $\hat{\rho}$, with $\mathrm{Tr}(\hat{\rho}) = 1$. Then pure states can be identified with those density matrices that are idempotent $\hat{\rho}^2 = \hat{\rho}$, i.e. with one–dimensional orthogonal projections.

Let us now consider the total system $T = Q \times C$. Later on we will define "experiment"as a coupling of C to Q. That coupling will take place within T.
First, let us consider statistical description, only after that we shall discuss dynamics and coupling of the two systems.
For the algebra $\mathcal{A}_t$ of observables of T we take the tensor product of algebras of observables of Q and C: $\mathcal{A}_t = \mathcal{A}_q \otimes \mathcal{A}_c$. It acts on the tensor product $\mathcal{H}_q \otimes \mathcal{H}_c = \oplus_{\alpha=1}^{m} \mathcal{H}_\alpha$, where $\mathcal{H}_\alpha \approx \mathcal{H}_q$. Thus $\mathcal{A}_t$ can be thought of as algebra of *diagonal* $m \times m$ matrices $A = (a_{\alpha\beta})$, whose entries are quantum operators: $a_{\alpha\alpha} \in \mathcal{A}_q$, $a_{\alpha\beta} = 0$ for $\alpha \neq \beta$.
The classical and quantum algebras are then subalgebras of $\mathcal{A}_t$; $\mathcal{A}_c$ is realized by putting $a_{\alpha\alpha} = f_\alpha I$, while $\mathcal{A}_q$ is realized by choosing $a_{\alpha\beta} = a\delta_{\alpha\beta}$.
Statistical states of $Q \times C$ are given by $m \times m$ diagonal matrices $\rho = \mathrm{diag}(\rho_1, \ldots, \rho_m)$ whose entries are positive operators on $\mathcal{H}_q$, with the normalization $\mathrm{Tr}(\rho) = \sum_\alpha \mathrm{Tr}(\rho_\alpha) = 1$. Tracing over C or Q produces the effective states of Q and C respectively: $\hat{\rho} = \sum_\alpha \rho_\alpha$, $p_\alpha = \mathrm{Tr}(\rho_\alpha)$.
Duality between observables and states is provided by the expectation value $< A >_\rho = \sum_\alpha \mathrm{Tr}(A_\alpha \rho_\alpha)$.

We consider now dynamics. Quantum dynamics, when no information is transferred from Q to C, is described by Hamiltonians H_α, that may depend on the actual state of C (as indicated by the index α). They may also depend explicitly on time. We will use matrix notation and write $H = \mathrm{diag}(H_\alpha)$. Now take the classical system. It is discrete here. Thus it can not have continuous time dynamics of its own.

Now we come to the crucial point – our main invention. A *coupling* of Q to C is specified by a matrix $V = (g_{\alpha\beta})$, with $g_{\alpha\alpha} = 0$. To transfer information from Q to C we need a non–Hamiltonian term which provides a completely positive (CP) coupling. We propose to consider couplings for which the evolution equation for observables and for states is given by the Lindblad form:

$$\dot{A} = i[H, A] + \mathcal{E}\left(V^\star A V\right) - \frac{1}{2}\{\Lambda, A\}, \tag{1}$$

$$\dot{\rho} = -i[H, \rho] + \mathcal{E}(V \rho V^\star) - \frac{1}{2}\{\Lambda, \rho\}, \tag{2}$$

where $\mathcal{E} : (A_{\alpha\beta}) \mapsto \mathrm{diag}(A_{\alpha\alpha})$ is the conditional expectation onto the diagonal subalgebra given by the diagonal projection, and

$$\Lambda = \mathcal{E}\left(V^\star V\right). \tag{3}$$

We can also write it down in a form not involving $\mathcal{E}$:

$$\dot{A} = i[H, A] + \sum_{\alpha \neq \beta} V^\star_{[\beta\alpha]} A V_{[\beta\alpha]} - \frac{1}{2}\{\Lambda, A\}, \tag{4}$$

with Λ given by

$$\Lambda = \sum_{\alpha \neq \beta} V^\star_{[\beta\alpha]} V_{[\beta\alpha]}, \tag{5}$$

and where $V_{[\alpha\beta]}$ denotes the matrix that has only one non–zero entry, namely $g_{\alpha\beta}$ at the α row and β column. Expanding the matrix form we have:

$$\dot{A}_\alpha = i[H_\alpha, A_\alpha] + \sum_\beta g^\star_{\beta\alpha} A_\beta g_{\beta\alpha} - \frac{1}{2}\{\Lambda_\alpha, A_\alpha\}, \tag{6}$$

$$\dot{\rho}_\alpha = -i[H_\alpha, \rho_\alpha] + \sum_\beta g_{\alpha\beta}\rho_\beta g^\star_{\alpha\beta} - \frac{1}{2}\{\Lambda_\alpha, \rho_\alpha\}, \tag{7}$$

where

$$\Lambda_\alpha = \sum_\beta g^\star_{\beta\alpha} g_{\beta\alpha}. \tag{8}$$

Again, the operators $g_{\alpha\beta}$ can be allowed to depend explicitly on time.

Following [4] we now define *experiment* and *measurement*:

Definition 1 *An* **experiment** *is a CP coupling between a quantum and a classical system. One observes then the classical system and attempts to learn from it about characteristics of state and of dynamics of the quantum system.*

Definition 2 *A* **measurement** *is an experiment that is used for a particular purpose: for determining values, or statistical distribution of values, of given physical quantities.*

The universe that we know, including us, the observers, can be considered as an "experiment". That point is discussed in [5].

3 The algorithm for events

The definition of experiment above is concerned with the *conditions* that define it. We will now describe the algorithm that simulates a typical *run* of a given experiment. That algorithm can be uniquely derived from the above formalism. One then gets the correct statistics by averaging over individual runs.

Let us first make a side but important remark. In practical situations it is rather easy to decide what constitutes Q, what constitutes C and how to write down the coupling. Then, if necessary, we enlarge Q, and we shift C towards more macroscopic and/or more classical. The new point of view that we propose allows us to consider our whole Universe as 'experiment' in which we are witnesses and participants of one particular run. Then one can ask: *what is the true C*? We do not know yet. Perhaps it has something to do with massless particles, with light, with photon detections. But perhaps we should not postpone any asking questions that are hard for a physicist: what is Knowledge and what is Mind?

Back to the main subject. It can be shown that there is a unique Markov process taking place in pure states of the total system that gives, after averaging over individual runs, time evolution of statistical states as described by Eq. (7). That process is piecewise deterministic – we call it PDP. Continuous evolution is interspersed with random jumps. Here it is:

PDP Algorithm 1 *Let us assume a fixed, sufficiently small, time step dt. Suppose that at time t the system is described by a quantum state vector ψ and a classical state α. Compute the scalar product $\lambda(\psi,\alpha) = <\psi, \Lambda_\alpha \psi>$. Toss dies and choose a uniform random number $p \in [0,1]$. Jump if $p < \lambda(\psi,\alpha)dt$. Otherwise not jump. When jumping, toss dies and change $\alpha \to \beta$ with probability $p_{\alpha\to\beta} = \|g_{\beta\alpha}\psi\|^2/\lambda(\psi,\alpha)$, and change $\psi \to g_{\beta\alpha}\psi/\|g_{\beta\alpha}\psi\|$. If not jumping, change*

$$\psi \to \frac{\exp\{-iH_\alpha dt - \frac{1}{2}\Lambda_\alpha dt\}\psi}{\|\exp\{-iH_\alpha dt - \frac{1}{2}\Lambda_\alpha dt\}\psi\|}, \quad t \to t + dt.$$

Repeat the steps.‡

For derivation and for a proof of uniqueness of the algorithm – see [5]. Our algorithm resembles that known in quantum optics as Wave Function Monte Carlo [7, 8, 9, 10, 11]. But there is an important difference: we did not guess our process. We derived it from M.H.A. Davis' mathematical theory of PDP processes [12]. We were also able to prove its uniqueness. That could not be achieved before. In fact, there is no uniqueness without an explicit introduction of a classical system. Ten years ago Diosi [13] (see

‡There are several methods available for efficient computation of the exponential for dt small enough – cf. Ref. [6].

also [14]) introduced "orthojump"process as a canonical solution to a master equation. His solution although canonical is not unique – unless one makes Hilbert spaces corresponding to different experimental situations orthogonal – as it is the case with our $\mathcal{H}_\alpha$-s.

We have mentioned in the beginning that our theory is falsifiable. Indeed, the PDP algorithm predicts time series of experimental events. They are changes of state of C. The continuous evolution between these events is affected be the coupling – it is *non–unitary* and *non–linear*. Its non–linearity depends on the copupling. Several examples have been already worked out. Some of them, including a SQUID–tank model, can be found in [15]. A cloud chamber model and its relation to GRW spontaneous localization models [16] have been worked out in [17].

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