1 Sim: Simplification by Annealing

Here I try to explain how to use “Sim”, to automatically simplify expressions.
Sim(ex) tries to minimize the length of the LaTeX representation of the expression "ex" by simulating annealing picking available transformations at random from the list TL of predefined transformations.
It is possible to direct the Simplificator to operate on a sub-expression by setting the parameter ops to a given list or tuple.
It is also possible to specify a substitution dictionary with subs_dic.
The use of these parameters is illustrated with an example from Monte Carlo simulation of a beta distribution by means of the rejection method.
A beta density with parameters $\alpha$ and $\beta$ (red curve below)
has a triangular envelope (the blue lines below)
provided that $\alpha > 2$ and $\beta > 2$.
The triangular envelope is easily computed with sage (betaT.sage)
We want a simplified formula for the upper vertex of the triangle that envelopes the beta density in terms of the parameters $\alpha$ and $\beta$.
The x coordinate of the upper vertex is $x_p$. The simplification is remarkable From length 564 to length 66 producing a beautiful formula.
Sim: Simplification by Annealing

\[ xp = \frac{1}{a_1^{-a_2} b_1^{-b_2} + 1} \]

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%typeset_mode True
%var a1, b1, a2, b2, a, c, y, z
load("http://omega.albany.edu:8008/sage/Sim.py")
load("http://omega.albany.edu:8008/sage/betaT.sage")

\[ p_0 + p_1 + p_2 \]

help(Sim)

Help on function Sim:

Sim(ex, Tmax=100, Tmin=1, steps=100, updates=0, energy=False, ops=(), subs_dic={})

Simplification of the expression ex by Simulated Annealing.
energy = length of the latex representation of ex.
If either ops or subs_dic is non empty then subexpression simplification and
substitution is applied by calling subSim.
See also “subSim”
Example:

sage: var('a b c d')
sage: e0 = sqrt((a^-3-b^-3)/(a-b)+a*b)
sage: Sim(e0)
sage: ex = integral(1/((a*x+b)^2*(c*x+c)^2),x)
sage: ex_sim = Sim(ex,energy=True)
sage: show([ex_sim,lenl(ex)])

sage: var('a a1 b x y')
sage: t = -2*a*b*x*y^2+3*b^2*x*y^2+(a^2*x-2*a*b*x+b^2*x)*y^2
sage: Sim(t,ops=(2,0),subs_dic={(a-b):a1},updates=5)

help(subSim)

Help on function subSim:

```
subSim(e, T_max=100, T_min=1, steps=100, updates=0, energy=False, ops=(), subs_dic={})
```

subexpression simplification and substitution.
It attempts to simplify e.op[ops] with 'Sim'.
After simplification it applies the substitutions given by the subs_dic dictionary.
It returns the original expression with the simplified subexpression and the substitutions.
Example:

sage: var('a a1 b x y')
sage: t = -2*a*b*x*y^2+3*b^2*x*y^2+(a^2*x-2*a*b*x+b^2*x)*y^2
sage: subSim(t,ops=(2,0),subs_dic={(a-b):a1})

x1 = Sim(xp)
x1

\[
(\alpha - 1)\alpha^2(\beta - 2)\beta - (\alpha - 2)\alpha(\beta - 1)^2\beta^2 + (\alpha - 1)\alpha^2(\beta - 2)^2 - 4(\alpha - 2)\alpha(\beta - 1)^2\beta - 4(\alpha - 1)^2\alpha
\]
dic1 = {(alpha -1) :a1,( alpha -2) :a2,( beta -1) :b1,( beta -2) :b2}

x2 = Sim(x1, subs_dic=dic1)
x2

\[
(\alpha + a_1^2)^2(\beta - 2)\beta + 4(\alpha + a_1^2)^2\beta^2 - (\alpha + a_1^2)^2(\alpha - 4a_1^2\alpha + 4a_1^2)\beta^2 - 4(\alpha + a_1^2)^2(\alpha - 4a_1^2\alpha + 4a_1^2)\beta
\]
x3 = Sim(x2, ops=(0,0), subs_dic=dic1)
x3

\[
(\alpha + a_1^2)^2(\beta - 2)\beta - (\alpha + a_1^2)^2\beta^2 - 4a_1^2\beta^2 - (\alpha + a_1^2)^2(\beta - 4a_1^2\alpha + 4a_1^2)\beta^2 - 4(\alpha + a_1^2)^2(\alpha - 4a_1^2\alpha + 4a_1^2)\beta
\]
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\[
x_4 = \text{Sim}(x_3, \text{ops}=(0,0,1), \text{subs_dic}=\text{dic1})
\]

\[
x_4 = \frac{a_1 \alpha^2 b_1 b_2}{a_1 a_2^2 b_1 b_2^\beta - a_1 a_2 b_1 b_2^\beta + 4 a_1 a_2^2 b_1 \beta - (4 (a_2^2 \alpha - a_2^\beta) b_1^\beta - (a_1^\alpha \alpha^2 - 4 a_1^\alpha \alpha^4 a_2^\beta) b_2^\beta)}
\]

\[
x_5 = \text{Sim}(x_4, \text{ops}=(0,0,3), \text{subs_dic}=\text{dic1})
\]

\[
x_5 = \frac{a_1^\alpha a_2^\beta b_1 b_2^\beta}{a_1 a_2^2 b_1 b_2^\beta - a_1 a_2 b_1 b_2^\beta + 4 a_1 a_2^2 b_1 \beta + (4 a_1^\alpha a_2^\beta b_1^\beta - 4 a_1 a_2^2 b_1^\beta b_2^\beta)}
\]

\[
x_6 = \text{Sim}(x_5, \text{ops}=(0,0), \text{subs_dic}=\text{dic1})
\]

\[
x_6 = \frac{a_1^\alpha a_2^\beta b_1 b_2^\beta}{a_1 a_2^2 b_1 b_2^\beta + a_1^\alpha a_2^\beta b_1^\beta - a_1 a_2^2 b_1^\beta}
\]

\[
x_7 = \text{Sim}(x_6, \text{ops}=(0,0), \text{subs_dic}=\text{dic1})
\]

\[
x_7 = \frac{a_1^\alpha a_2^\beta b_1 b_2^\beta}{a_1 a_2^2 b_1 b_2^\beta + a_1 a_2^\beta b_1^\beta b_2^\beta}
\]

\[
x_8 = \text{Sim}(x_7, \text{ops}=(0,0), \text{subs_dic}=\text{dic1})
\]

\[
x_8 = \frac{a_1^\alpha a_2^\beta b_1 b_2^\beta}{a_1^- a_2^- a_2^\beta b_1 b_2^- b_2^+ 1}
\]

\[
\text{dic1} \cdot \text{update} \{(\text{-alpha+1}): -a_1, (\text{-beta+2}): -b_2\}
\]

\[
\text{dic1} \{-\alpha + 1 : -a_1, \alpha - 1 : a_1, \beta - 1 : b_1, -\beta + 2 : -b_2, \alpha - 2 : a_2, \beta - 2 : b_2\}
\]

\[
x_9 = \text{Sim}(\text{i}x_8, \text{subs_dic}=\text{dic1})
\]

\[
x_9 = a_1^- a_2^- a_2^- b_1^\beta b_2^- b_2^+ 1
\]

# Final formula: for xp

\[
\text{xp_simplified} = 1/x_9
\]

\[
\text{xp_simplified} = 1/\left(1/\left(\frac{x_1^- a_2^- a_2^\beta b_1 b_2^- b_2^+ 1}{a_1^- a_2^- a_2^- b_1^\beta b_2^- b_2^+ 1}\right)\right)
\]
len1(xp)
len1(xp_simplified)
  564
  66