CELLULAR AUTOMATA AND INFERENCE

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Abstract

Stephen Wolfram popularized elementary one-dimensional cellular automata in his book, *A New Kind of Science*. Among many remarkable things, he proved that one of these cellular automata was a Universal Turing Machine. Such cellular automata can be interpreted in a different way by viewing them within the context of the formal manipulation rules from probability theory. Bayes's Theorem is the most famous of such formal rules.

As a prelude, we recapitulate Jaynes's presentation of how probability theory generalizes classical logic by using *modus ponens* as a canonical example. We give an alternative demonstration augmenting and complementing Jaynes's derivations by emphasizing formal operations arising from Boolean Algebra. Then, we show the complementary roles played in inferential arguments of this kind by Bayes's Theorem and joint probability tables.

A good explanation for all of this is afforded by the expansion of any particular logic function via the *disjunctive normal form* (DNF). The DNF expansion is a useful heuristic emphasized in this exposition because such expansions point out where relevant 0s should be placed in the joint probability tables for logic functions involving any number of variables. The DNF makes it easy to see where 0s should be placed in joint probability tables for cellular automata. It is then a short leap to generalize deduction using logic functions as well as deterministic cellular automata.

We proceed to a straightforward exercise relying on Boolean Algebra, Bayes's Theorem, and joint probability tables in extrapolating to Wolfram's cellular automata. Cellular automata are seen as purely deductive systems, just like classical logic, which probability theory is then able to generalize. Wolfram emphasized the correct, but pessimistic view, that prediction in deductive systems is almost impossible. Nevertheless, the quintessential feature of probability theory and inferential systems, as first clearly demonstrated by Jaynes in applications to statistical mechanics, is the capability of predicting some future macro-events if one is willing to discard information about micro-events. If cellular automata are taken as test-beds for complicated, detailed, ontological models of the world, then, more optimistically, prediction about future events might take place. Some speculation is offered on one technical approach on how best to discard micro-information within maximum entropy type models.